BAL BHARATI PUBLIC SCHOOL, PITAMPURA, DELHI – 110034

SUBJECT: MATHEMATICS **CLASS: X**

CHAPTERS: Real Numbers

Polynomials

Pair of Linear Equations in Two Variables

WEEK :1

Number of blocks -2

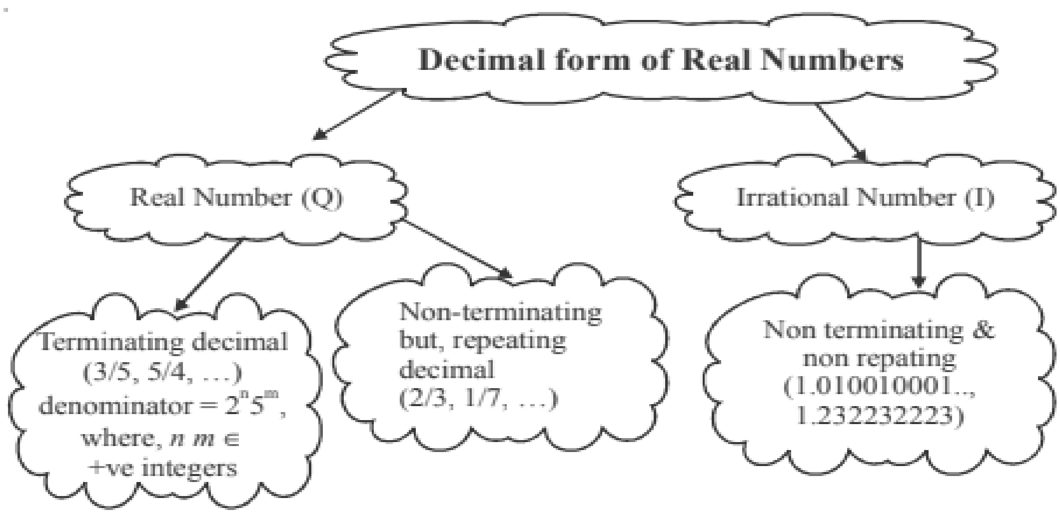
GUIDELINES FOR THE STUDENTS:-

- Note the format of the answers carefully.
- Refer to NCERT and revise the subtopic along with the intext questions related to the revision lesson.
- Recapitulation will be held in the subsequent classes through random questioning.

SUBTOPICS: -

REAL NUMBERS-

KEY POINTS



Fundamental theorem of Arithmetic-

Every composite number can be expressed factorised as a product of primes and this factorisation is unique apart from the order in which the prime factors occur.

IMPORTANT QUESTIONS ON REAL NUMBERS

Prove that $\sqrt{7}$ is an irrational number.

Sol. Let us assume, to the contrary, that $\sqrt{7}$ is a rational number.

Then, there exist co-prome positive integers a and b such that

$$\sqrt{7} = \frac{a}{b}, \qquad b \neq 0$$

So, $a = \sqrt{7}b$

Squaring both sides, we have

$$a^2 = 7b^2$$

$\Rightarrow$ 7 divides a^2 $\Rightarrow$ 7 divides a

So, we can write

$$a = 7c, \qquad \text{(where c is any integer)}$$

Putting the value of $a = 7c$ in (i), we have

$$49c^2 = 7b^2 \quad \Rightarrow \quad 7c^2 = b^2$$

It means 7 divides b^2 and so 7 divides b.

So, 7 is a common factor of both a and b which is a contradiction.

So, our assumption that $\sqrt{7}$ is rational is wrong.

Hence, we conclude that $\sqrt{7}$ is an irrational number.

Que Show that 5-$\sqrt{3}$ is an irrational number.

Sol. Let us assume that 5-$\sqrt{3}$ is rational.

So, 5-$\sqrt{3}$ may be written as

$5-\sqrt{3} = \frac{p}{q}$, where p and q are integers, having no common factor except 1 and $q \neq 0$.

$$\Rightarrow 5 - \frac{p}{q} = \sqrt{3} \qquad \Rightarrow \qquad \sqrt{3} = \frac{5q - p}{q}$$

Since $\frac{5q-p}{q}$ is a rational number which is a contradiction.

$\therefore \sqrt{3}$ is also a rational number which is a contradiction.

Thus, our assumption is wrong.

Hence, 5 - $\sqrt{3}$ is an irrational number.

Que The decimal representation of $\frac{6}{1250}$ will terminate after how many places of decimal?

Sol. $\frac{6}{1250} = \frac{3}{625} = \frac{3}{5^4} \times \frac{2^4}{2^4} = \frac{48}{(5 \times 2)^4} = \frac{48}{10^4} = 0.0048$

This representation will terminate after 4 decimal places.

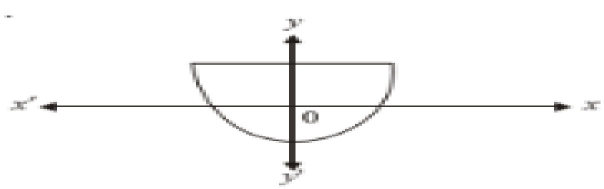
POLYNOMIALS-
KEY POINTS

Graph of different types of polynomials:

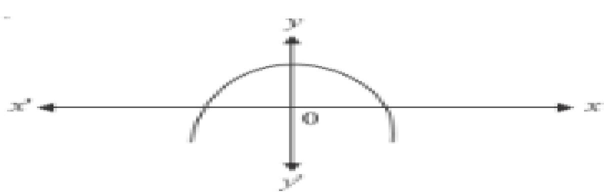
Linear Polynomial : The graph of a linear polynomial $ax + b$ is a straight line, intersecting x-axis at one point.

Quadratic Polynomial:

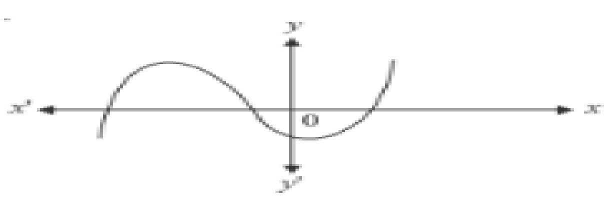
(*i*) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open upwards like U, if a > 0 and intersect x-axis at maximum two distinct points.



(*ii*) Graph of a quodratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open downwards like ∩, if a < 0 and intersect x-axis at maximum two distinct points.



(*iii*) Polynomial and its graph : In general a polynomial $p(x)$ of degree n crosses the x-axis at most n points.



(i) If one zero of a quadratic polynomial $p(x)$ is negative of the other, then coefficient of x is 0.

(ii) If zeroes of a quadratic polynomial $p(x)$ are reciprocal of each other, then coefficient of x^2 = constant term.

Relationship between zeros and coefficients of a polynomial

If α and β are zeros of $p(x) = ax^2 + bx + c$ $(a \neq 0)$, then

Sum of zeros $= \alpha + \beta = -\frac{b}{a}$

Product of zeros $= \alpha\beta = \frac{c}{a}$

If α, β are zeros of a quadratic polynomial $p(x)$, then

$p(x) = k\,[x^2 - (\text{sum of zeros})\,x + \text{product of zeros}]$

$\Rightarrow\ p(x) = k\,[x^2 - (\alpha + \beta)x + \alpha\beta\,]$; where k is any non-zero real number.

IMPORTANT QUESTIONS ON POLYNOMIALS-

Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

Answer:

α and β are the zeroes of the polynomial $2x^2 - 3x + 1$

$$\Rightarrow \qquad \alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{2} = \frac{3}{2}$$

$$\alpha\beta = \frac{c}{a} = \frac{1}{2}$$

Now, zeroes of the required polynomial are 3α and 3β

$$\Rightarrow \qquad S = 3\alpha + 3\beta = 3(\alpha + \beta) = 3\left(\frac{3}{2}\right) = \frac{9}{2}$$

$$\Rightarrow \qquad P = (3\alpha)(3\beta) = 9(\alpha\beta) = 9 \times \frac{1}{2} = \frac{9}{2}$$

Now, required polynomial is $x^2 - Sx + p$

$$= x^2 - \frac{9}{2}x + \frac{9}{2} = \frac{k}{2}(2x^2 - 9x + 9)$$, where k be any constant.

Que **If one zero of polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other, find the value of a.**

Sol. Let one zero of the given polynomial be a.

Then, the other zero is $\frac{1}{a}$

$\therefore$ *Product of zeros* $= a \times \frac{1}{a} = 1$

But, as per the given polynomial product of zeros = $\frac{6a}{a^2+9}$

$\therefore \quad \frac{6a}{a^2+9} = 1 \quad \Rightarrow \quad a^2 + 9 = 6a$

$\Rightarrow \quad a^2 - 6a + 9 = 0 \quad \Rightarrow \quad (a-3)^2 = 0$

$\Rightarrow \quad a - 3 = 0 \quad \Rightarrow \quad a = 3$

Form the polynomial whose zeroes are $3 + \sqrt{3}$, $3 - \sqrt{3}$.

Solution:

The zeroes are $3 + \sqrt{3}$, $3 - \sqrt{3}$. ∴ Sum of the zeroes = $(3 + \sqrt{3}) + (3 - \sqrt{3}) = 6$ ∴

Product of the zeroes = $(3 + \sqrt{3})(3 - \sqrt{3})$

$= 9 - (\sqrt{3})^2$

$= 9 - 3$

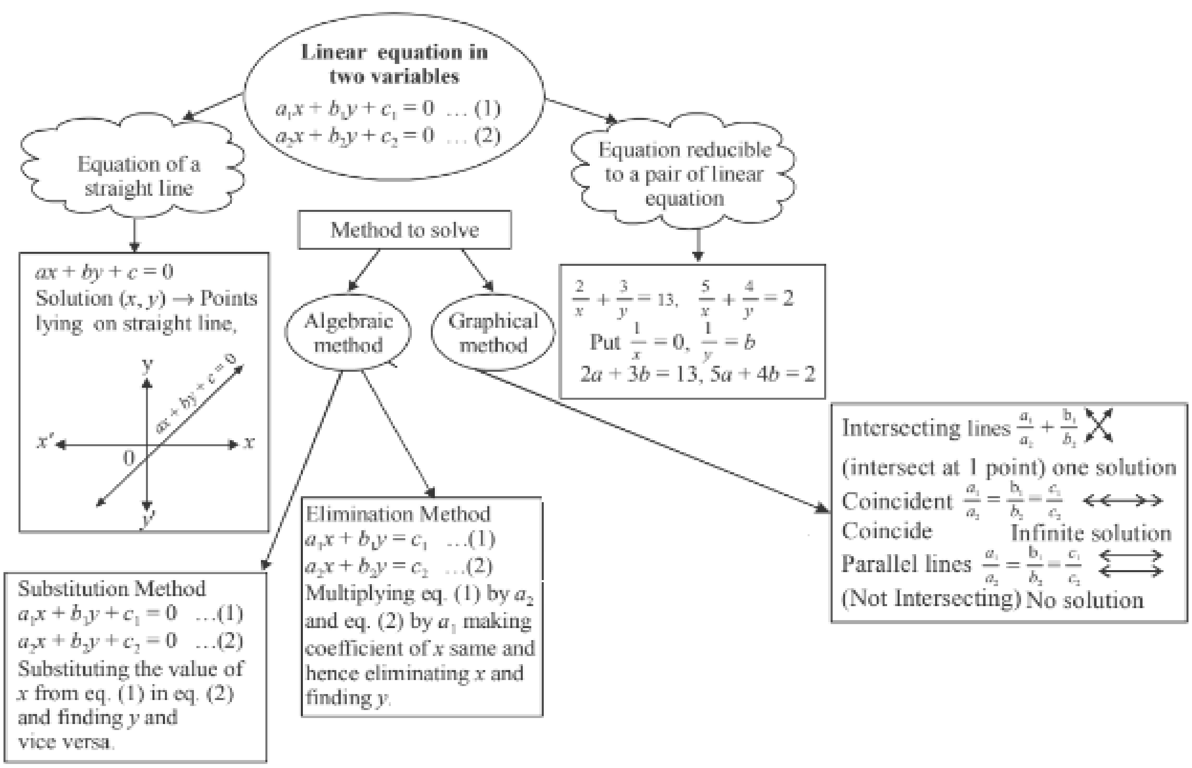
$= 6$

∴ The required polynomial is

x^2 - (sum of the zeroes) x + Product of the zeroes ⇒ $x^2 - (6)x + (6)$ Þ $x^2 - 6x + 6$

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES-

KEY POINTS



IMPORTANT QUESTIONS ON LINEAR EQUATION IN TWO VARIABLES-

For what value of k, the pair of equations 4x − 3y = 9, 2x + ky = 11 has no solution?

Solution:

We have, 4x − 3y = 9 and 2x + ky = 11

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \text{(No solution)}$$

$$\frac{4}{2} = \frac{-3}{k} \neq \frac{9}{11} \qquad \Rightarrow \qquad 2 = \frac{-3}{k}$$

$$2k = -3 \qquad \therefore \qquad k = \frac{-3}{2}$$

To verify-

$$\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{-3}{-3/2} \neq \frac{9}{11}$$

verified, therefore the pair of equation has no solution for

$$k = \frac{-3}{2}$$

Also, try to find the area bound between two lines and the two axes.

Check graphically whether the pair of equations 3x − 2y + 2 = 0 and $\frac{3}{2}$x − y + 3 = 0, is consistent. Also find the coordinates of the points where the graphs of the equations meet the Y-axis.

Solution:

$3x - 2y + 2 = 0$

$\Rightarrow \quad y = \frac{3x+2}{2}$

x	0	2	−2
y	1	4	−2

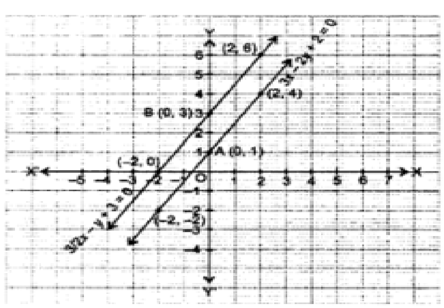
(0, 1), (2, 4), (−2, −2)

$\frac{3}{2}x - y + 3 = 0$

$y = \frac{3}{2}x + 3$

x	0	2	−2
y	3	6	0

(0, 3), (2, 6), (−2, 0)



By plotting the points and joining them, the lines do not intersect anywhere, i.e., they are parallel.

Therefore given pair of equations is not consistent, i.e., inconsistent.

The equation 3x − 2y + 2 = 0 meets the Y-axis at A(0,1).

The equation $\frac{3}{2}$x − y + 3 = 0 meets the Y-axis at B(0, 3).

Solve for x and y

$$\frac{6}{x-1} - \frac{3}{y-2} = 1; \quad \frac{5}{x-1} + \frac{1}{y-2} = 2 \quad \text{Where } x \neq 1, y \neq 2$$

Sol. Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$

The given equations become

$$6p - 3q = 6 \quad \ldots(i)$$

$$5p + q = 2 \quad \ldots(ii)$$

Multiply equation (ii) by 3 and add in equation (i)

$$\begin{array}{l} 15p + \;\; 3q = \;\; 6 \\ \;\;6p - \;\; 3q = \;\; 1 \\ \hline \;\; 21p \;\; = \;\; 7 \end{array} \qquad \Rightarrow \quad p = \frac{7}{21} = \frac{1}{3}$$

Putting this value in equation (i) we get

$$6 \times \frac{1}{3} - 3q = 1 \quad \Rightarrow \quad 2 - 3q = 1 \quad \Rightarrow \quad 3q = 1, \quad \Rightarrow \quad q = \frac{1}{3}$$

Now, $\frac{1}{x-1} = p = \frac{1}{3} \quad \Rightarrow \quad x - 1 = 3 \quad \Rightarrow \quad x = 4$

$\frac{1}{y-2} = q = \frac{1}{3} \quad \Rightarrow \quad y - 2 = 3 \quad \Rightarrow \quad y = 5$

Hence, $x = 4$ and $y = 5$

Revision Assignment

1. Two positive integers a and b can be written as $a = x^3y^2$ and $b = xy^3$. x, y are prime numbers. Find the LCM (a, b) and the HCF(a,b) . 2
2. Prove that $\sqrt{5}$ is irrational and hence prove that $(2\sqrt{5} - 3)/7$ is an irrational number. 3
3. Find the largest number which divides 414 and 608, leaving remainders 6 and 8 respectively. 3
4. a)

   After how many places the decimal expansion of $\frac{13497}{1250}$ will terminate? 1

   b)

   Without actual performing the long division, find if $\frac{395}{10500}$ will have terminating or non terminating (repeating decimal expansion.) 1

5. If HCF of 144 and 180 is expressed in the form 13m – 3, find the value of m. 3

6. In a seminar, the no. of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required if in each room the some the same number of participants are to be seated and all of the them being of the the same subject. (HOTS) 3

7.

If n is a natural number, then $25^{2n} - 9^{2n}$ is always divisible by :

(i) 16 (ii) 34

(iii) both 16 or 34 (iv) None of these

1

8. Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeros equal to half to their product. 3

9 ..

If α and β are zeros of the polynomial $t^2 - t - 4$, form a quadratic polynomial whose zeros are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. 3

10.

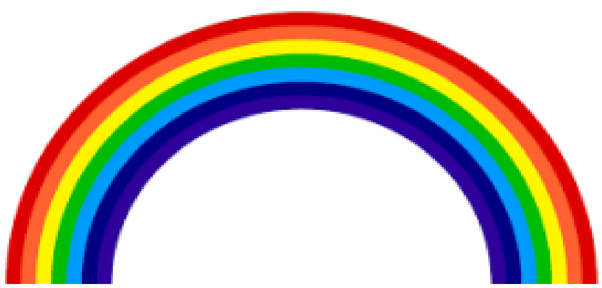
If one zero of $p(x) = 4x^2 - (8k^2 - 40k)x - 9$ is negative of the other, find values of k. 2

11.

Obtain zeros of $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3}$ and verify relation between its zeroes and coefficients. 3

12.

Rainbow is an arch of colours that is visible in the sky, caused by the refraction and dispersion of the light after rain or other water droplets in the atmosphere. The colours of the rainbow are generally said to be red, orange, yellow, green, blue, indigo and violet.

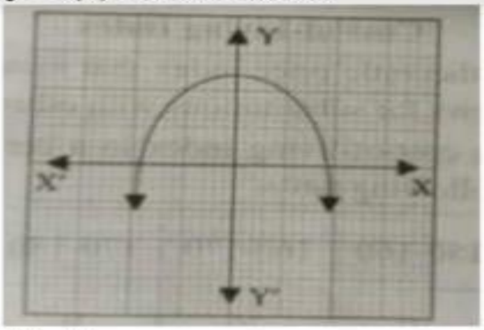


Each colour of the rainbow makes a parabola. We know that for any quadratic polynomial $ax^2 + bx + c = 0$, $a \neq 0$, the graph of the corresponding equation $y = ax^2 + bx + c$ has one of the two shapes either open upwards like ∪ or open downwards like ∩ depending on whether $a > 0$ or $a < 0$. These curves are called parabolas.

i. A rainbow is represented by the quadratic polynomial $x^2 + (a+1)x + b$ whose zeroes are 2 and -3, then
(a) a = 7, b = -1 (b) a = 5, b = -1 (c) a = 2, b = -6 (d) a = 0, b = -6

ii. If α and β are the zeroes of the rainbow represented by the polynomial $x^2 - 2x - 15$, then the polynomial whose zeroes are 2α and 2β is
(a) $x^2 + 4x + 60$ (b) $x^2 - 4x + 60$
(c) $x^2 + 4x - 60$ (d) $x^2 - 4x - 60$

iii. The graph of a rainbow $y = f(x)$ is shown below



The number of zeroes of f(x) is

(a) 0 (b) 1 (c) 2 (d) 3

iv. If graph of the rainbow does not intersect x-axis but intersects y-axis in one point, then the number of zeroes of the polynomial is equal to

(a) 0 (b) 1 (c) 0 or 1 (d) none of these

v. The representation of a rainbow is a quadratic polynomial. The sum and product of the zeroes are 3 and -2 respectively. The polynomial is

(a) $k(x^2 - 2x - 3)$, for some real k (b) $k(x^2 - 5x - 9)$, for some real k

(c) $k(x^2 - 3x - 2)$, for some real k (d) $k(x^2 - 3x + 2)$, for some real k

13.

Find a quadratic polynomial one of whose zeros is $(3+\sqrt{2})$ and the sum of its zeroes is 6. **2**

14.

For what value of k, the following system of equations will be inconsistent

$$kx + 3y = k - 3$$
$$12x + ky = k$$

2

15.

Find the value of k for which the following pair of linear equations have infinitely many solutions. [2]

$2x + 3y = 7$, $(k + 1)x + (2k - 1)y = 4k + 1$

2

16.

Solve for x and y

$$\frac{5}{x+y} + \frac{1}{x-y} = 2$$
$$\frac{15}{x+y} - \frac{5}{x-y} = -2$$

3

17.

For what values of a and b the following pair of linear equations have infinite number of solutions? (CBSE)

$$2x + 3y = 7$$
$$a(x + y) - b(x - y) = 3a + b - 2$$

3

18.

For what value of p the pair of linear equations $(p+2)x-(2p+1)y=3(2p-1)$ and $2x-3y=7$ has a unique solution. 2

19. Solve-

$$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2 \text{ and } \frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$$ 3

20.

Solve for x and y

$$0.4x+0.3y=1.7$$
$$0.7x-0.2y=0.8$$ **2**

21.

Draw the graphs of following equations:

$2x-y=1$; $x+2y=13$

Find the solution of the equations from the graph and shade the triangular region formed by the lines and the y-axis.

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