BAL BHARATI PUBLIC SCHOOL, PITAMPURA

Class -9 Mathematics

TOPIC: SURFACE AREAS AND VOLUMES (part 3)

Week: 14th Dec to 18th Dec

Number of blocks: 4

Subtopics:

- **Surface area of cone**
- **Surface area of a sphere**
- **Surface area of hemisphere**

NOTE- Students can download the NCERT textbook using the following link: - http://ncert.nic.in/textbook/textbook.htm?hemh1=0-16

Learning Outcomes:

Each student will be able to:

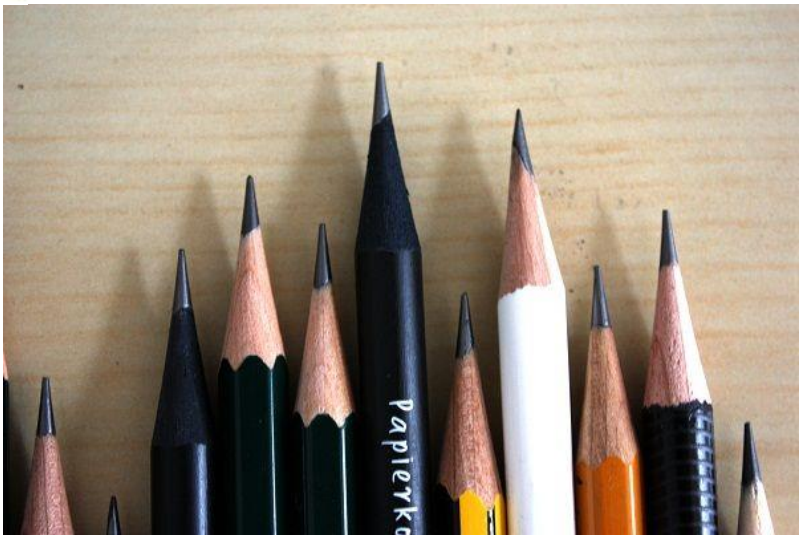
- Visualise a cone in its 2-D form in order to calculate the surface area.
- Visualise a sphere in order to calculate surface area.

Teaching Aids Used: Presentation by screen sharing, Digital Board, White-board and Marker, MS- Word You tube videos, Quizlet, Kahoot

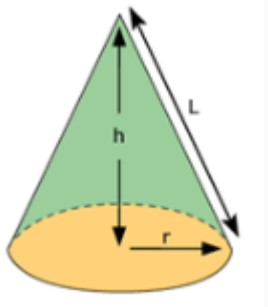
LESSON DEVELOPMENT

BLOCK 1

Students will be asked to give examples of conical objects from day to day life.



Now surface area of Right circular cone will be introduced



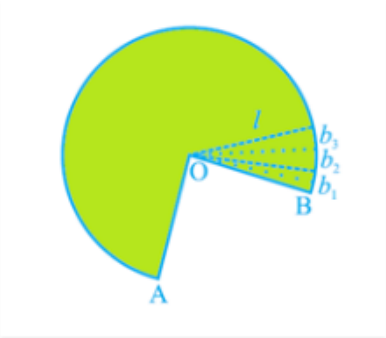
The base is a simple circle and we know that area of a circle is given as:

Area of a circle=πr²

Where ***r*** is the base radius of the cone

Area of the curved surface:

Now if open the curved top and cut into small pieces, so that each cut portion is a small triangle, whose height is the slant height *l* of the cone.



Now the area of each triangle =1/2× base of each triangle × *l*.

∴Area of the curved surface = sum of the areas of all the triangles

$$= \frac{1}{2} \times b_1 \times l + \frac{1}{2} \times b_2 \times l + \frac{1}{2} \times b_3 \times l + \ldots\ldots\ldots + \frac{1}{2} \times b_n \times 1$$

$$= \frac{1}{2} l\,(b_1 + b_2 + b_3 + \ldots\ldots + b_n)$$

$$= \frac{1}{2} l\,(curved surface)$$

From the figure, we know that, the curved surface is equivalent to the perimeter of the base of the cone.

The circumference of the base of the cone = 2πr

∴ Area of the curved surface = ½ × l ×2πr

Area of the curved surface= πrl

Total Surface Area of a Cone = Area of the circular base + Area of the curved surface

Total Surface Area of a Cone = $\pi r^2 + \pi rl$

s

Total surface area of a cone = πr (l + r)

Following questions will be discussed during class:

Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24m.

Solution:

Slant height (l) = 21 m

Diameter of base = 24 m

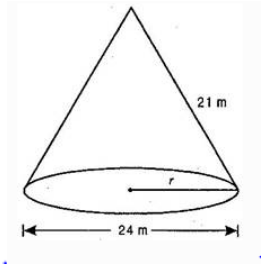
$\therefore$ Radius of base (r) = $\frac{24}{2}$ m = 12 m

$\therefore$ Total surface area of the cone = $\pi r(l + r)$

$= \frac{22}{7} \times 12 \times (21 + 12)$

$= \frac{22}{7} \times 12 \times 33 = \frac{8712}{7}$

$= 1244\frac{4}{7}$ m^2.



Question:

Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find

(i) radius of the base and

(ii) Total surface area of the cone.

Solution:

Curved surface area of cone = 308 cm^2

$\Rightarrow \pi rl = 308$

$\Rightarrow \frac{22}{7} \times r \times 14 = 308$

$\Rightarrow r = \frac{308 \times 7}{14 \times 22}$

$\Rightarrow r = 7$ cm

(ii) Total surface area of the cone

= Curved surface area + Area of circular base

$= 308 + \pi r^2$

$= 308 + \frac{22}{7} \times (7)^2$

= 462 cm^2

Question:

What length of tarpaulin 3m wide will be required to make conical tent of height 8m and base radius 6m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20cm (use π = 3.14).

Solution:

For conical tent

Given,

h = 8 m, r = 6 m

$\therefore l = \sqrt{r^2 + h^2}$

$= \sqrt{(6)^2 + (8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ m

$\therefore$ Width surface area = $\pi rl = 3.14 \times 6 \times 10 = 188.4\ m^2$

Width of tarpaulin = 3 m

$\therefore$ Length of tarpaulin = $\frac{188.4}{3} = 62.8$ m

Extra length of the material required = 20 cm = 0.2 m

$\therefore$ Actual length of tarpaulin required = 62.8 m + 0.2 m = 63 m.

Question:

The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white - washing its curved surface at the rate of Rs. 210 per 100 m^2.

Solution:

Slant height (l) = 25 m

Base diameter = 14 m

$\therefore$ Base radius (r) = $\frac{14}{2}$ m = 7 m

$\therefore$ Curved surface area of the tomb = $\pi rl = \frac{22}{7} \times 7 \times 25$ = 550 m^2

$\therefore$ Cost of white-washing the curved surface of the tomb at the rate of Rs. 210 per 100 m^2

$$= \text{Rs. } \frac{210}{100} \times 550 = \text{Rs. } 1155.$$

Practice Question 1:

A conical tent is 10 m high and the radius of its base is 24 m. Find
(i) Slant height of the tent.
(ii) Cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is ₹ 70.

Q2.

A Joker's cap is in the form of a right circular cone of base radius 7cm and height 24cm. Find the area of the sheet required to make 10 such caps.

PRACTICE QUESTIONS:

1. **Find the volume of cone of radius r/2 and height '2h'.**
2. **Find the capacity in litres of a conical vessel having height 8 cm and slant height 10 cm.**
3. **A right angled A ABC with sides 3 cm, 4 cm and 5 cm is revolved about the fixed side of 4 cm. find the volume of the solid generated. Also, find the total surface area of the solid.**

LESSON DEVELOPMENT

Block 2:

Recapitulation of concept taught on first day and following question will be discussed:

Question:

A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1m. If the outer side of each of the cones is to be painted and the cost of painting is Rs.12 per m^2, What will be the cost of painting all these cones? (use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$).

Solution:

Given, Base diameter of the cone = 40 cm

$\therefore$ Base radius (r) = $\frac{40}{2}$ cm = 20 cm = $\frac{20}{100}$ m = 0.2m

Height (h) = l m

$\therefore l = \sqrt{r^2 + h^2} = \sqrt{(0.2)^2 + (1)^2} = \sqrt{0.04 + 1} = \sqrt{1.04}$ = 1.02 m (approximately).

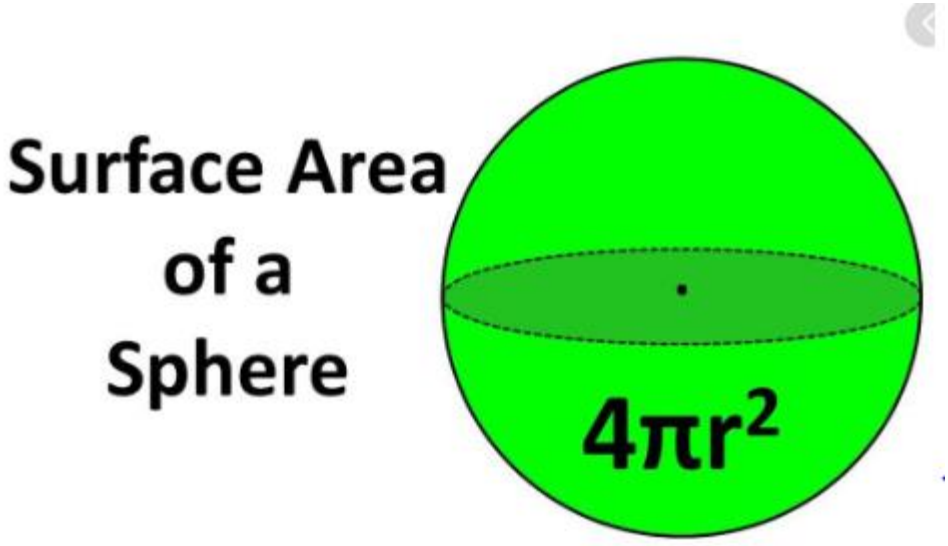
$\therefore$ Curved surface area = $\pi rl = 3.14 \times 0.2 \times 1.02 = 0.64056\ m^2$

$\therefore$ Curved surface area of 50 cones = $0.64056 \times 50\ m^2 = 32.028\ m^2$

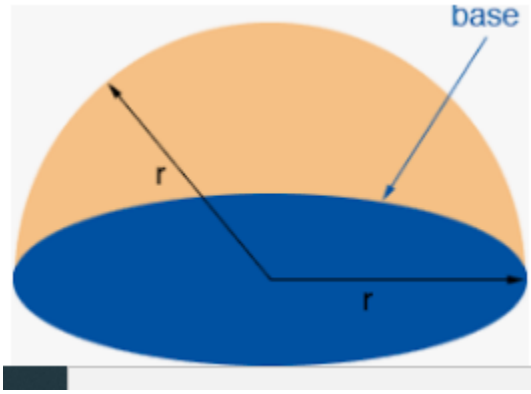
$\therefore$ Cost of painting all these cones = $32.028 \times 12 = 384.336$ = Rs. 384.34 (approximately).

Following images will be shown to children and name of its shape will be asked:

After discussions, Surface area of Sphere will be introduced.



If we cut sphere from the center , we get Hemisphere



hemisphere	Lateral surface area	Total surface area
r	$2\pi r^2$	$3\pi r^2$

Following questions will be discussed during class:

QUESTION:

Find the surface area of a sphere of radius: (i) 10.5 cm (ii) 5.6 cm (iii) 14cm

Solution:

(i) r = 10.5 cm $\therefore$ Surface area = $4\pi r^2 = 4 \times \frac{22}{7} \times (10.5)^2 = 1386\ cm^2$.

(ii) r = 5.6 cm

$\therefore$ Surface area = $4\pi r^2 = 4 \times \frac{22}{7} \times (5.6)^2 = 394.24\ cm^2$.

(iii) r = 14 cm

$\therefore$ Surface area = $4\pi r^2 = 4 \times \frac{22}{7} \times (14)^2 = 2464\ cm^2$.

QUESTION:

Find the surface area of a sphere of diameter: (i) 14 cm (ii) 21cm (iii) 3.5m

Solution:

(i) Diameter - 14 cm

$\therefore$ Radius (r) = $\frac{14}{2}$ cm = 7 cm

$\therefore$ Surface area = $4\pi r^2 = 4 \times \frac{22}{7} \times (7)^2 = 616\ cm^2$.

(ii) Diameter = 21 cm

$\therefore$ Radius (r) = $\frac{21}{2}$ cm

$\therefore$ Surface area = $4\pi r^2 = 4 \times \frac{22}{7} \times \left(\frac{21}{2}\right)^2 = 1386\ cm^2$.

(iii) Diameter = 3.5 cm

$\therefore$ Radius (r) = $\frac{3.5}{2}$ cm = 1.75 cm

$\therefore$ Surface area = $\pi r^2 = 4 \times \frac{22}{7} \times (1.75)^2 = 38.5\ cm^2$.

QUESTION:

The radius of a spherical balloon increases from 7cm to 14cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Solution:

Case I. r = 7 cm

Surface area = $4\pi r^2 = 4 \times \frac{22}{7} \times (7)^2 = 616\ cm^2$.

Case II. r = 14 cm

$\therefore$ Surface area = $4\pi r^2 = 4 \times \frac{22}{7} \times (14)^2 = 2464\ cm^2$

$\therefore$ Ratio of surface areas of the balloon = 616 : 2464

$= \frac{616}{2464} = \frac{1}{4} = 1 : 4$

PRACTICE QUESTIONS:

1. **A shot-put is a metallic sphere of radius 4.9 cm. If the density of the metal is 7.8 g/cm^3. Find the mass of the shot-put.**

2. **A spherical ball is divided into two equal halves. If the curved surface area of each half is 56.57 cm^2, find the volume of the spherical ball. [use π = 3.14]**

LESSON DEVELOPMENT

Block 3:

Recapitulation of concept taught on previous day and following question will be discussed:

QUESTION:

A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of Rs.16 per 100cm^2.

Solution:

Inner diameter = 10.5 cm

$\therefore$ Inner radius (r) = $\frac{10.5}{2}$ cm = 5.25 cm

$\therefore$ Inner surface area = $2\pi r^2 = 2 \times \frac{22}{7} \times (5.25)^2 = 173.25\ cm^2$

$\therefore$ Cost of tin-plating at the rate of Rs. 16 per 100 cm^2

$= 173.25 \times \frac{16}{100}$ = Rs. 27.72

QUESTION:

Find the radius of a sphere whose surface area is 154cm^2.

Solution:

Let the radius of the sphere be r cm.

Surface area = 154 cm^2

$\Rightarrow 4\pi r^2 = 154$

$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = \frac{154 \times 7}{4 \times 22}$

$\Rightarrow r^2 = \frac{49}{4}$

$\Rightarrow r = \sqrt{\frac{49}{4}} \Rightarrow r = \frac{7}{2} = 3.5$ cm

Hence the radius of the sphere is 3.5 cm.

QUESTION:

The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Solution:

Let the diameter of the earth be 2r.

Then diameter of the moon = $\frac{1}{4}(2r) = \frac{r}{2}$

$\therefore$ Radius of the earth = $\frac{2r}{2} = r$

and, Radius of the moon = $\frac{1}{2}\left(\frac{r}{2}\right) = \frac{r}{4}$

$\therefore$ Surface area of the earth = $4\pi r^2$

and, Surface area of the moon = $4\pi \left(\frac{r}{4}\right)^2 = \frac{1}{4}\pi r^2$

Ratio of their surface areas = $\frac{\text{Surface area of the moon}}{\text{Surface area of the earth}}$

$$= \frac{\frac{1}{4}\pi r^2}{4\pi r^2} = \frac{1}{16}$$

$\therefore$ Ratio of their surface areas = 1 : 16

QUESTION:

A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

Solution:

Inner radius of the bowl = 5 cm
Thickness of steel = 0.25 cm
∴ Outer radius of the bowl = 5 + 0.25 = 5.25 cm

Outer curved surface area of bowl = $2\pi r^2$

= $2 \times 22/7 \times 5.25 \times 5.25$

= $173.25\ cm^2$

QUESTION:

A right circular cylinder just encloses a sphere of radius r. Find

(i) surface area of the sphere

(ii) curved surface area of the cylinder

(iii) ratio of the areas obtained in (i) and (ii).

Solution:

(i) Surface area of the sphere = $4\pi r^2$

(ii) **For cylinder**

Radius of the base = r

Height = 2r

∴ Curved surface area of the cylinder = $2\pi (r)(2r) = 4\pi r^2$

(iii) Ratio of the areas obtained in (i) and (ii)= $\frac{\text{Surface area of the sphere}}{\text{Curved surface area of the cylinder}}$

$= \frac{4\pi r^2}{4\pi r^2} = \frac{1}{1}$

= 1 : 1.

Link for reference:

https://www.youtube.com/watch?v=tCvjX-PSFj0

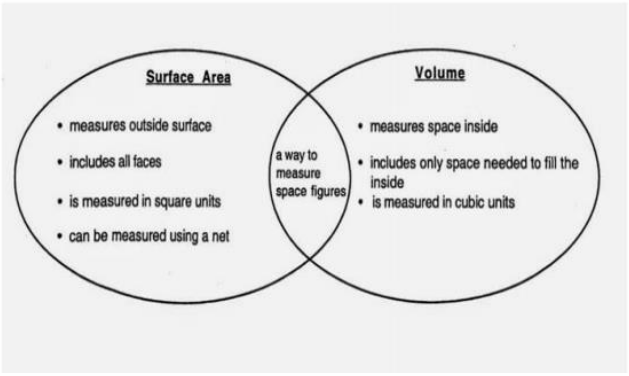
https://www.youtube.com/watch?v=kQfvbXBoIN4 **(Exercise 13.4 Q9)**

Block 4:

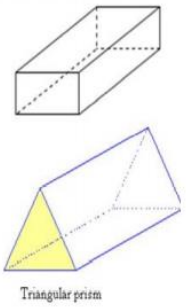
LESSON DEVELOPMENT

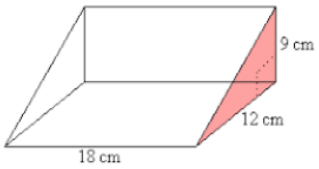
CASE STUDY

Case 1



- Definition:
  - A three dimensional figure with 2 congruent polygon bases and rectangular sides.
- Finding the Surface Area:
  - Add up the areas of all of the sides.





Find the surface area of the given solid

If we calculate the surface area and volume of a polyhedron

(i)surface area will be greater than the volume (ii)surface area will be less than the volume

(iii)Surface area will be equal to the volume (iv) both cannot be compared.

The surface area of a rectangular prism of length' L', breadth 'B' and height 'H' is ____
If a cylinder has the same radius and height as a sphere, then the ratio of their volume is (i) 1:1 (ii) 3:4 (iii) 4:3 (iv) 3:2

PRACTICE QUESTIONS FROM EXEMPLAR:

Question 1:

If the radius of a sphere is 2r, then its volume will be

(a) $\frac{4}{3}\pi r^3$ (b) $4\pi r^3$ (c) $\frac{8\pi r^3}{3}$ (d) $\frac{32}{3}\pi r^3$

Question2:

A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. The radius of the sphere is

(a) 4.2 cm

(b) 2.1 cm

(c) 2.4 cm

(d) 1.6 cm

Question3:

The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. The ratios of the surface areas of the balloon in the two cases is

(a) 1 : 4

(b) 1 : 3

(c) 2 : 3

(d) 2 : 1

Write whether true or false and justify your answer

Question4:

The volume of a sphere is equal to two-third of the volume of a cylinder whose height and diameter are equal to the diameter of the sphere.

Question5:

If the radius of a right circular cone is halved and height is doubled, then volume will remain unchanged,

Question6:

In a right circular cone, height, radius and slant height do not always be sides of a right triangle,

Question7:

The volume of the largest right circular cone that can be fitted in a cube whose edge is 2r equals to the volume of a hemisphere radius r.

Question8:

A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is 1: 2: 3.