BAL BHARATI PUBLIC SCHOOL, PITAMPURA, DELHI – 110034

SUBJECT: MATHEMATICS- CLASS: X

WEEK: 1st to 5th November
SUBJECT: MATHEMATICS
CLASS: X
NO. OF BLOCKS: 4
TOPIC: Triangles(Part- 2)

Guidelines:-

Dear students, kindly refer to the following link of notes/video links from the chapter
https://ncert.nic.in/ncerts/l/jemh106.pdf

SUBTOPICS:-

- Areas of Similar Triangles
- Pythagoras Theorem
- Converse of Pythagoras Theorem

INSTRUCTIONAL AIDS:-Presentation by screen sharing, offline whiteboard, online whiteboard, YouTube links, E-lesson.

LEARNING OUTCOMES:-

Each Student will be able

- find the relation between the ratio of areas of Similar triangles and its sides
- state and apply Pythagoras Theorem
- state and apply Converse of Pythagoras Theorem

Block 1

Lesson Development

Areas of Similar Triangles

You have learnt that in two similar triangles, the ratio of their corresponding sides is the same. Do you think there is any relationship between the ratio of their areas and the ratio of the corresponding sides? You know that area is measured in square units. So, you may expect that this ratio is the square of the ratio of their corresponding sides. This is indeed true and we shall prove it in the next theorem.
Theorem: The ratio of the areas of two similar triangles is equal to the square of the ratio of

their corresponding sides.

Activity

Let us verify the theorem through this manual activity

Materials Required

Chart paper, construction box, colored pens, a pair of scissors, fevicol

Procedure

1. Take a chart paper and cut a ΔABC with AB = 6cm, BC = 6cm, CA = 6cm.
2. Mark 5 points P_1 , P_2 ,…. , P_5 at a distance of 1cm each on side AB and Q_1 , Q_2 ,… .., Q_5 at a distance of 1cm each on side AC as shown in fig. (I).

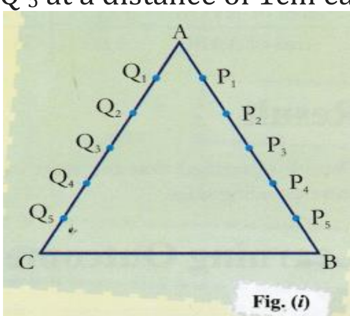


Fig. (*i*)

3. Join $P_1 Q_1$, $P_2 Q_2$, …………, $P_5 Q_5$ as shown in fig. (ii).

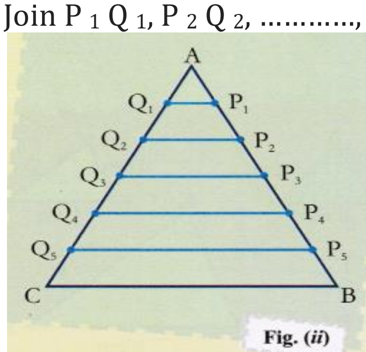


Fig. (*ii*)

4. Draw lines parallel to AC from P_1 , P_2 , P_5 and also draw lines parallel to AB from the points Q_1 , Q_2 , ……., Q_5 as shown in fig. (iii).

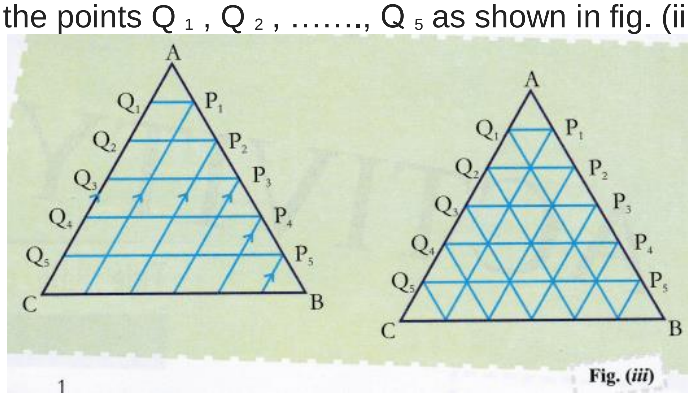
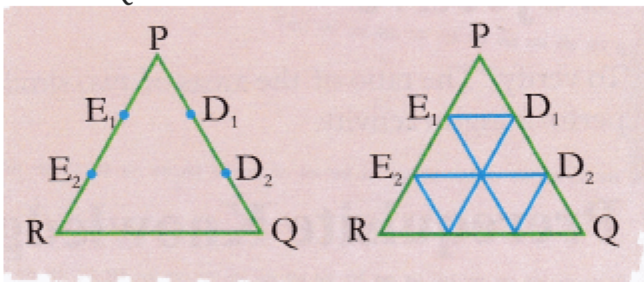


Fig. (*iii*)

5. Thus ΔABC is divided into 36 smaller triangles and all are similar to each other and of equal area.

6. Construct a ΔPQR with PQ = $\frac{1}{2}$of AB, PR = $\frac{1}{2}$of AC and QR = $\frac{1}{2}$of BC i.e. 3cm each on another chart paper.
7. Mark D $_1$, D $_2$ and E $_1$, E $_2$ on sides PQ and PR respectively.
8. Repeat steps 3 and 4.
9. Thus ΔPQR is divided into 9 smaller similar triangles equal in area.



Observation

1. area of ΔABC = area of 36 smaller Δ's
2. area of ΔPQR = area of 9 smaller Δ's
3. $\frac{PQ}{AB} = \frac{3}{6} = \frac{1}{2} = \frac{PR}{AC}$
4. $\frac{Area \quad of \quad \Delta PQR}{Area \quad of \quad \Delta ABC} = \frac{PQ^2}{AB^2}$

   = [9 smaller Δ's / 36 smaller Δ's = $\frac{1}{4}$= (1/2) ^{2}

Result

Thus, it is verified that the ratio of the areas of two similar triangles is equal to the ratio of the square of their respective sides.

Ex 6.4 Question 1

Let ΔABC ~ ΔDEF and their areas be, respectively, 64 cm² and 121 cm². If EF = 15.4 cm, find BC.

Solution:

We have $\Delta ABC \sim \Delta DEF$

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So, $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{CA}{FD}$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \quad \frac{64}{121} = \frac{(BC)^2}{(15.4)^2} = \frac{BC^2}{237.16}$$

$$\Rightarrow \quad 121BC^2 = 237.16 \times 64$$

$$\Rightarrow \quad BC^2 = \frac{15178.24}{121} = 125.44$$

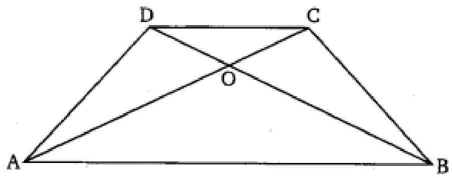
$$\Rightarrow \quad BC = \sqrt{125.44} = \mathbf{11.2\ cm.}$$

Question 2.

Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.

Solution:

In the figure below, a trapezium ABCD is shown, in which AB || DC and AB = 2DC. Its diagonals interest each other at the point O.



In ΔAOB and ΔCOD

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\angle OAB = \angle OCD$ [Corresponding angles]

$\therefore \Delta AOB \sim \Delta COD$ [By AA similarity]

$$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{DC^2} = \frac{(2 \times DC)^2}{DC^2}$$

$$= \frac{4 \times DC^2}{DC^2} = \frac{4}{1}$$

Hence, $ar(\Delta AOB) : ar(\Delta COD) = \mathbf{4 : 1}$

Refer to the following links to understand more about applications of this theorem

https://www.examfear.com/free-video-lesson/Class-10/Maths/Triangles/part-35/Triangles_Part_35_(Theorem_Ratio_of_Sides_of_triangle).htm

https://www.examfear.com/free-video-lesson/Class-10/Maths/Triangles/part-36/Triangles_Part_36_(Theorem_Ratio_of_Area_of_triangle).htm

https://www.examfear.com/free-video-lesson/Class-10/Maths/Triangles/part-37/Triangles_Part_37_(Example_Ratio_of_Area_of_triangle).htm

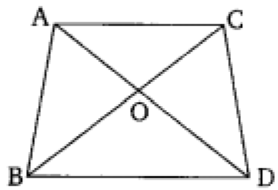
Block 2

Lesson Development

The following questions from Exercise 6.4 will be discussed during the session

Question 3.

In the given figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that: $\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$



Question 4.

If the areas of two similar triangles are equal, prove that they are congruent.

Question 5.

D, E and F are respectively the mid-points of sides AB, BC and CA of △ABC. Find the ratio of the areas of △DEF and △ABC.

Question 6.

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Question 7.

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Assignment: Do Q8 and Q9 from Ex 6.4 in your Math Register.

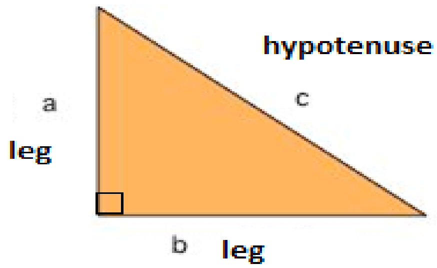
Block 3

Lesson Development

Pythagoras Theorem

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

(Proof of the theorem will be discussed in the class)



The Pythagorean Theorem tells us that the relationship in every right triangle is:

$$a^2+b^2=c^2$$

Activity

Objective

To verify Pythagoras theorem by performing an activity.

Procedure

Step 1: Paste a sheet of white paper. On this paper, draw a right-angled triangle ABC, right angled at C. Let the lengths of the sides AB, BC and CA be c, a and b units respectively (see Figure 10.1).

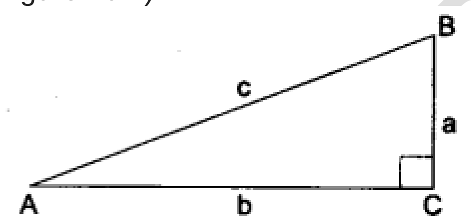


Fig. 10.1

Step 2: Make four exact copies of the right-angled ΔABC on the other sheet of paper. Also, construct a square with each side measuring c units.

Step 3: Cut these four triangles and the square and arrange them as shown in Figure 10.2.

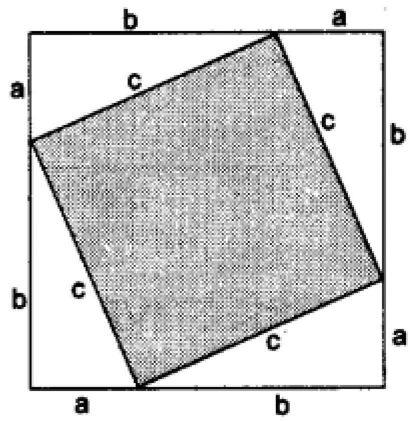


Fig. 10.2

Observations and Calculations

We observe that by the combination of the square and the four triangles, a new square is formed which clearly has each side equal to (a+b) units. Then, the
area of the large square formed = area of the square with side c + 4 (area of ΔABC)
i.e., $(a+b)^2 = c^2 + 4(\frac{1}{2} \times a \times b)$ [∴ area of ΔABC = ½ (a x b)]
=> $(a^2 + b^2 + 2ab) = c^2 + 2ab$
=> $a^2 + b^2 = c^2$.
So, the square of the hypotenuse of right-angled ΔABC is equal to the sum of the squares of the other two sides.

Result

Pythagoras' theorem is verified.

Converse of the Pythagorean Theorem

The theorem states that if the square of the length of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

(Proof of the above theorem is deleted)

Ex 6.5 Question 1.

Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.
(i) 7 cm, 24 cm, 25 cm
(ii) 3 cm, 8 cm, 6 cm

Solution:

Solution:

(i) $(7)^2 + (24)^2 = 49 + 576 = 625 = (25)^2$

Therefore, these are the sides of a right triangle. The length of its hypotenuse is **25 cm.**

(ii) $(3)^2 + (6)^2 = 9 + 36 = 45 \neq 64 \neq (8)^2$

Therefore, these are not the sides of a right triangle.

Question 2.

PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that $PM^2 = QM \times MR$

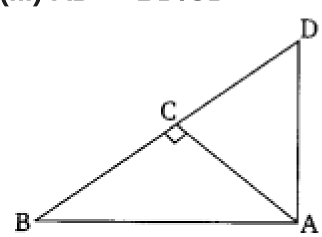
Question 3.

In the given figure, ABD is a triangle right angled at A and AC i. BD. Show that

(i) $AB^2 = BC.BD$

(ii) $AC^2 = BC.DC$

(iii) $AD^2 = BD.CD$



Question 5.

ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, Prove that ABC is a right triangle.

Question 7.

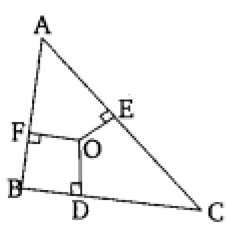
Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Question 8.

In the given figure, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.



Assignment: Do questions Q1 (iii) ,(iv), Q4 , Q6 from Exercise 6.5 in your Math register.

Refer to the following links to understand more about Pythagoras theorem and its converse

https://www.examfear.com/free-video-lesson/Class-10/Maths/Triangles/part-38/Triangles_Part_38_(Proof_Pythagoras_Theorem).htm
https://www.examfear.com/free-video-lesson/Class-10/Maths/Triangles/part-41/Triangles_Part_41_(Example_Pythagoras_Theorem).htm
https://www.examfear.com/free-video-lesson/Class-10/Maths/Triangles/part-42/Triangles_Part_42_(Example_Pythagoras_Theorem).htm

Block 4

Lesson Development

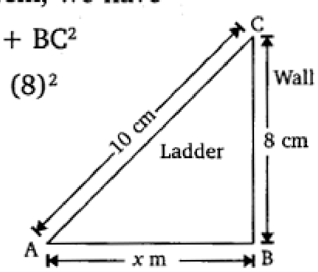
Ex6.5 Question9.

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Solution:

Let the distance from the wall AB be x m. Then by Pythagoras' Theorem, we have

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow (10)^2 = x^2 + (8)^2$$
$$\Rightarrow 100 - 64 = x^2$$
$$\Rightarrow 36 = x^2$$
$$\Rightarrow 6 = x$$
$$\Rightarrow x = 6$$



Hence, the distance of ladder from the wall is **6 m**.

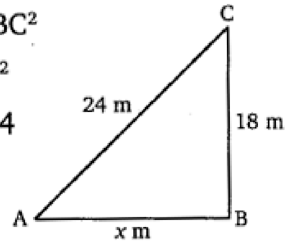
Ex 6.5 Class 10- Question 10.

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Solution:

Let the distance of the stake from the base be x m.

Then in right ΔABC by Pythagoras' Theorem,



$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad (24)^2 = x^2 + 18^2$$

$$\Rightarrow \quad 576 = x^2 + 324$$

$$\Rightarrow \quad 576 - 324 = x^2$$

$$\Rightarrow \quad x^2 = 252$$

$$\Rightarrow \quad x = \sqrt{252} = 6\sqrt{7} \text{ m.}$$

Hence, the stake should be driven at a distance of $\mathbf{6\sqrt{7}}$ **m** from the base of the pole.

Question 11.

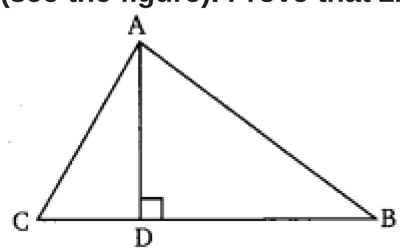
An airplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another airplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Question 13.

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Question 14.

The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3CD (see the figure). Prove that $2AB^2 = 2AC^2 + BC^2$.



Question 15.

In an equilateral triangle ABC, D is a point on side BC, such that BD = $\frac{1}{3}$BC. Prove that $9AD^2 = 7AB^2$.

Question 16.

In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Refer to the following links to understand more about Pythagoras theorem's applications

https://www.examfear.com/free-video-lesson/Class-10/Maths/Triangles/part-43/Triangles_Part_43_(Example).htm

https://www.examfear.com/free-video-lesson/Class-10/Maths/Triangles/part-44/Triangles_Part_44_(Example_Pythagoras_Theorem).htm

https://www.examfear.com/free-video-lesson/Class-10/Maths/Triangles/part-

45/Triangles_Part_45_(Example_Pythagoras_Theorem).htm

Assignment: Do questions Q12 ,Q17 in your Math Register.

AAC Activity

Construct three polynomials (linear, Quadratic and Cubic) with number 3 as one of the zero of the polynomials.

Summary:

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (Pythagoras Theorem).

If in a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

1. Refer to the following links to practice more questions
a) https://www.learncbse.in/triangles-cbse-class-10-extra-questions-with-solutions/
b) From Khan Academy Assignments
https://www.khanacademy.org/math/in-in-grade-10-ncert
c) www.examfear.com
d) http://www.ei-india.com/mindspark-math (free trial for 60 days)