BAL BHARATI PUBLIC SCHOOL, PITAMPURA, DELHI – 110034

SUBJECT:- MATHEMATICS

CHAPTER10:- CIRCLES(PART-2)

Week : 12th October to 16th October'2020

Number of blocks : 4

Subtopics :

- Equal chords
- Distance of equal chords from the centre
- Perpendicular from the centre to a chord

Link for the chapter : http://ncert.nic.in/textbook/textbook.htm?jemh1=3-15

Learning Outcomes:

Each student will be able to :

- Define equal chords of a Circle
- Prove that equal chords of a circle (or of congruent circles) are equidistant from the centre
- Prove that Chords equidistant from the centre of a circle are equal in length
- Apply these theorems in different problems
- Prove that the angle subtended by an arc at the centre is double the angle subtended by it at any point of the remaining circle.

Teaching Aids Used :

Presentation of E-lesson, PDF of NCERT textbook, YouTube videos by screen sharing, white board and marker or register and pen using laptop/mobile camera, digital board, google Jamboard etc.

GUIDELINES:

Dear Students
Kindly read the content given below and view the links shared for better understanding.

Solve the given questions in the yellow register provided in the notebook set.

Link for the chapter : http://ncert.nic.in/textbook/textbook.htm?jemh1=3-15

DAY 1

INTRODUCTION ACTIVITY

Students will recall the following theorems

- The perpendicular from the centre of a circle to a chord bisects the chord and vice-versa.
- It two chords of a circle are equal, then their corresponding arcs are congruent and conversely, if two arcs are congruent, then their corresponding chords are equal.
- Congruent arcs/chords of a circle subtend equal angles at the centre.

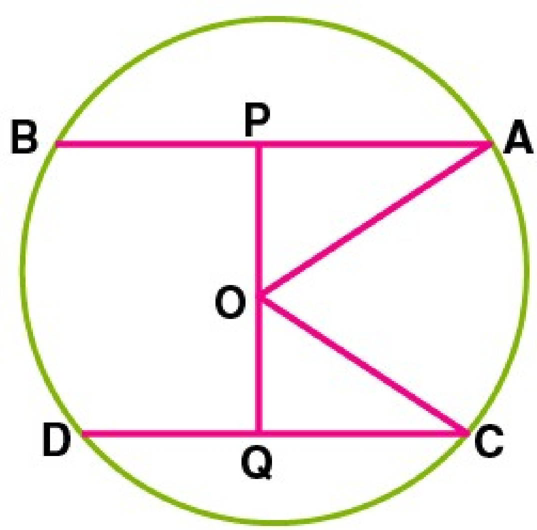
NOTE : THEOREM 10.5 AND 10.10 ARE DELETED FROM THE SYLLABUS

LESSON DEVELOPMENT

Equal Chords and their Distances from the Centre.

The length of the perpendicular from a point to a line is the distance of the line from the point.

Theorem: Equal chords of a circle (or of congruent circles) are **equidistant from the centre** (or centres).

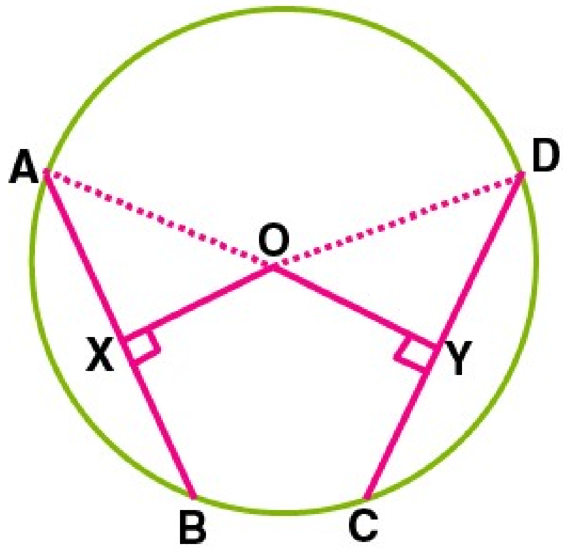


Given, AB = CD, O is the centre.

⇒ OP = OQ

Chords equidistant from the centre are equal

Theorem 10.7 : Chords **equidistant** from the centre of a circle are **equal in length.**

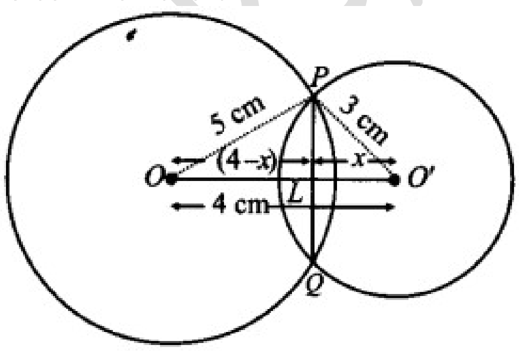


Given OX = OY

⇒ AB = CD

Ex 10.4

Question 1.
Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.
Solution:
We have two intersecting circles with centres at O and O' respectively. Let PQ be the common chord.
∵ In two intersecting circles, the line joining their centres is perpendicular bisector of the common chord.



∴∠OLP = ∠OLQ = 90° and PL = LQ
Now, in right ∆OLP, we have
$PL^2 + OL^2 = OP^2$
⇒ $PL^2 + (4 – x)^2 = 5^2$
⇒ $PL^2 = 5^2 – (4 – x)^2$
⇒ $PL^2 = 25 - 16 – x^2 + 8x$
⇒ $PL^2 = 9 – x^2 + 8x$ …(i)
Again, in right ∆O'LP,

$PL^2 = PO'^2 – LO'^2$
$= 3^2 – x^2 = 9 – x^2$ …(ii)
From (i) and (ii), we have
$9 – x^2 + 8x = 9 – x^2$
⇒ 8x = 0
⇒ x = 0
⇒ L and O' coincide.
∴ PQ is a diameter of the smaller circle.
⇒ PL = 3 cm
But PL = LQ
∴ LQ = 3 cm
∴ PQ = PL + LQ = 3cm + 3cm = 6cm
Thus, the required length of the common chord = 6 cm.

ASSIGNMENT:

Do the following work from NCERT book in the yellow register
1.Solved example number 2.
2. Revise the theorems.

Links for the reference :

https://youtu.be/euGC01eSKqM
https://youtu.be/Y5ti_gctLZE
https://youtu.be/axpuujYtjxo

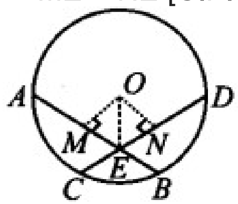
DAY 2

LESSON DEVELOPMENT

Ex 10.4

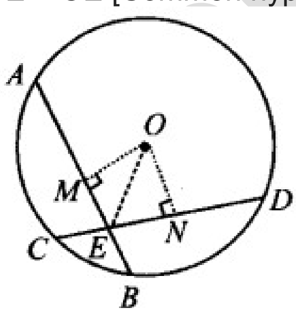
Question 2.
If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.
Solution:
Given: A circle with centre O and equal chords AB and CD intersect at E.
To Prove: AE = DE and CE = BE
Construction : Draw OM ⊥ AB and ON ⊥ CD.
Join OE.
Proof: Since AB = CD [Given]
∴ OM = ON [Equal chords are equidistant from the centre]
Now, in ∆OME and ∆ONE, we have
∠OME = ∠ONE [Each equal to 90°]
OM = ON [Proved above]

OE = OE [Common hypotenuse]
∴ ΔOME ≅ ΔONE [By RHS congruence criteria]
⇒ ME = NE [C.P.C.T.]



Adding AM on both sides, we get
⇒ AM + ME = AM + NE
⇒ AE = DN + NE = DE
∵ AB = CD ⇒ $1/2\ \mathrm{AB} = 1/2\mathrm{DC}$
⇒ AM = DN
⇒ AE = DE …(i)
Now, AB – AE = CD – DE
⇒ BE = CE …….(ii)
From (i) and (ii), we have
AE = DE and CE = BE

Question 3.
If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.
Solution:
Given: A circle with centre O and equal chords AB and CD are intersecting at E.
To Prove : ∠OEA = ∠OED
Construction: Draw OM ⊥ AB and ON ⊥ CD.
Join OE.
Proof: In ΔOME and ΔONE,
OM = ON
[Equal chords are equidistant from the centre]
OE = OE [Common hypotenuse]

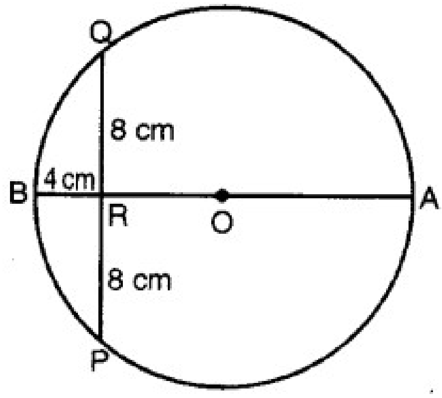


∠OME = ∠ONE [Each equal to 90°]
∴ ΔOME ≅ ΔONE [By RHS congruence criteria]
⇒ ∠OEM = ∠OEN [C.P.C.T.]
⇒ ∠OEA = ∠OED

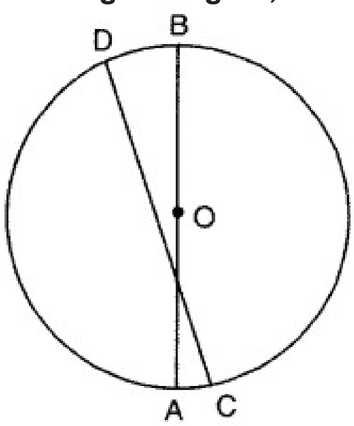
Extra Questions for practice

1.The given figure shows a circle with centre O in which a diameter AB bisects the chord PQ at the point R. If PR = RQ = 8 cm and RB = 4 cm, then find the

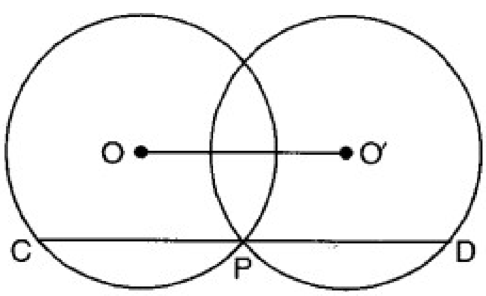
radius of the circle.



2. In the given figure, O is the centre of the circle, then compare the chords.



3.Two circles whose centres are O and O' intersect at P. Through P, a line parallel to OO', intersecting the circles at C and D is drawn as shown in the figure. Prove the CD = 200'.



Links for the reference :

https://youtu.be/axpuujYtjxo

https://youtu.be/axpuujYtjxo

ASSIGNMENT

1.Learn the theorems and understand their application.

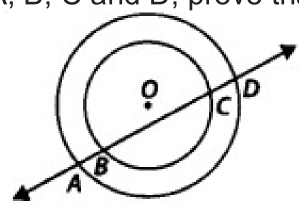
2.Revise all the questions and attempt the practice questions.

DAY 3

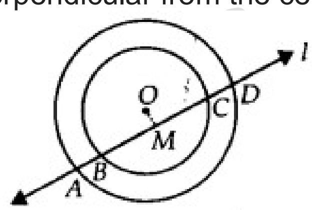
LESSON DEVELOPMENT

Ex 10.4

Question 4.
If a line intersects two concentric circles (circles with the same centre) with centre 0 at A, B, C and D, prove that AB = CD (see figure).



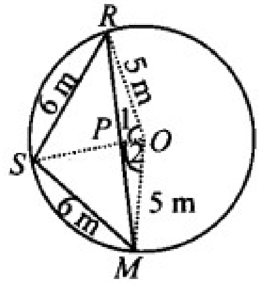
Solution:
Given : Two circles with the common centre O.
A line D intersects the outer circle at A and D and the inner circle at B and C.
To Prove : AB = CD.
Construction:
Draw OM ⊥ l.
Proof: For the outer circle,
OM ⊥ l [By construction]
∴ AM = MD …(i)
[Perpendicular from the centre to the chord bisects the chord]



For the inner circle,
OM ⊥ l [By construction]
∴ BM = MC …(ii)
[Perpendicular from the centre to the chord bisects the chord]
Subtracting (ii) from (i), we have
AM – BM = MD – MC
⇒ AB = CD

Question 5.
Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?
Solution:
Let the three girls Reshma, Salma and Mandip be positioned at R, S and M

respectively on the circle with centre O and radius 5 m such that
RS = SM = 6 m [Given]

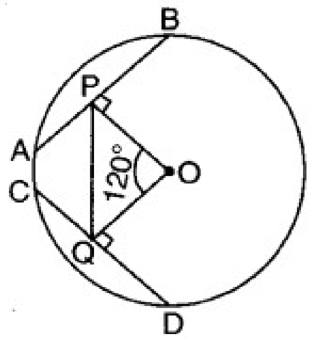


Equal chords of a circle subtend equal angles at the centre.
∴ ∠1 = ∠2
In ∆POR and ∆POM, we have
OP = OP [Common]
OR = OM [Radii of the same circle]
∠1 = ∠2 [Proved above]
∴ ∆POR ≅ ∆POM [By SAS congruence criteria]
∴ PR = PM and
∠OPR = ∠OPM [C.P.C.T.]
∵∠OPR + ∠OPM = 180° [Linear pair]
∴∠OPR = ∠OPM = 90°
⇒ OP ⊥ RM
Now, in ∆RSP and ∆MSP, we have
RS = MS [Each 6 cm]
SP = SP [Common]
PR = PM [Proved above]
∴ ∆RSP ≅ ∆MSP [By SSS congruence criteria]
⇒ ∠RPS = ∠MPS [C.P.C.T.]
But ∠RPS + ∠MPS = 180° [Linear pair]
⇒ ∠RPS = ∠MPS = 90°
SP passes through O.
Let OP = x m
∴ SP = (5 – x)m
Now, in right ∆OPR, we have
$x^2 + RP^2 = 5^2$
$RP^2 = 5^2 - x^2$
In right ∆SPR, we have
$(5 - x)^2 + RP^2 = 6^2$
⇒ $RP^2 = 6^2 - (5 - x)^2$ ……..(ii)
From (i) and (ii), we get
⇒ $5^2 - x^2 = 6^2 - (5 - x)^2$
⇒ $25 - x^2 = 36 - [25 - 10x + x^2]$
⇒ – 10x + 14 = 0
⇒ 10x = 14 ⇒ x = 1410 = 1.4
Now, $RP^2 = 5^2 - x^2$
⇒ $RP^2 = 25 - (1.4)^2$
⇒ $RP^2 = 25 - 1.96 = 23.04$
∴ RP = $\sqrt{23.04}$ = 4.8
∴ RM = 2RP = 2 x 4.8 = 9.6
Thus, distance between Reshma and Mandip is 9.6 m.

Practice **Questions**

1. If ABC is an equilateral triangle inscribed in a circle and P be any point on the minor arc BC which does not coincide with B or C, prove that PA is angle bisector of ∠BPC .

2. In the given figure, AB and CD are two equal chords of a circle with centre O. OP and OQ are perpendiculars on chords AB and CD respectively. If ∠ POQ = 120°, find ∠APQ .



ASSIGNMENT:

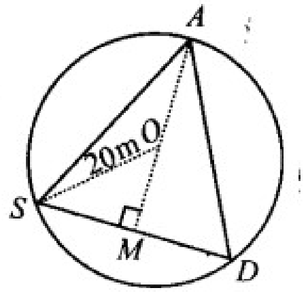
Revise all the questions and attempt the practice questions.

DAY 4

LESSON DEVELOPMENT

Ex 10.4

Question 6.
A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.
Solution:
Let Ankur, Syed and David are sitting at A, S and D respectively in the circular park with centre O such that AS = SD = DA
i. e., ∆ASD is an equilateral triangle.
Let the length of each side of the equilateral triangle be 2x.
Draw AM ⊥ SD.
Since ∆ASD is an equilateral triangle.
∴ AM passes through O.
⇒ SM = $1/2$ SD = $1/2$ (2x)
⇒ SM = x



Now, in right ΔASM, we have
$AM^2 + SM^2 = AS^2$ [Using Pythagoras theorem]
$\Rightarrow AM^2 = AS^2 - SM^2 = (2x)^2 - x^2$
$= 4x^2 - x^2 = 3x^2$
$\Rightarrow AM = \sqrt{3}x$ m
Now, $OM = AM - OA = (\sqrt{3}x - 20)$m
Again, in right ΔOSM, we have
$OS^2 = SM^2 + OM^2$ [using Pythagoras theorem]
$20^2 = x^2 + (\sqrt{3}x - 20)^2$
$\Rightarrow 400 = x^2 + 3x^2 - 40\sqrt{3}x + 400$
$\Rightarrow 4x^2 = 40\sqrt{3}x$
$\Rightarrow x = 10\sqrt{3}$ m
Now, $SD = 2x = 2 \times 10\sqrt{3}$ m $= 20\sqrt{3}$ m
Thus, the length of the string of each phone $= 20\sqrt{3}$ m

RELATED LINK :

https://youtu.be/OYpjf-C045U

Maths Lab Manual – Verify that the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

OBJECTIVE

To verify that the angle subtended by an arc of a circle at the centre is double the angle subtended by it at an point on the remaining part of the circle.

Material Required:

1. Coloured drawing sheets
2. Cardboard
3. Geometry box
4. White paper
5. Adhesive
6. Transparent sheet

7. Cutter/Scissors

Prerequisite Knowledge

1. All the basic knowledge related to the circle.
2. Angle subtended by an arc.

Theory

1. The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle. The fixed point is called the centre of the circle, the line segment joining the centre and any point on the circle is called radius of circle.
2. A line segment joining two points on the circle is called a chord of the circle.
3. A chord which passes through the centre of the circle is called a diameter of the circle.
4. The length of the complete circle is called its circumference.
5. A piece of a circumference of circle between two points is called an arc.

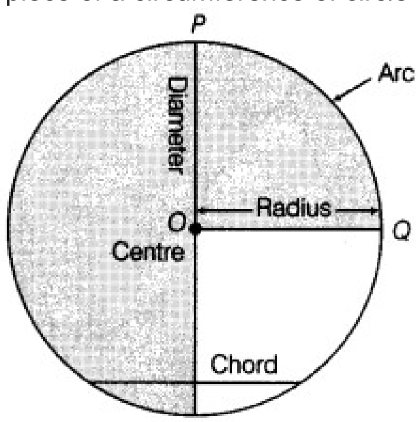


Fig. 23.1

6. Angle subtended by an arc of a circle
   Let us draw a circle with centre at O and AB be its arc. Here, ∠AOB is the angle subtended by arc AB ($\bar{AB}$) at the centre of the circle.

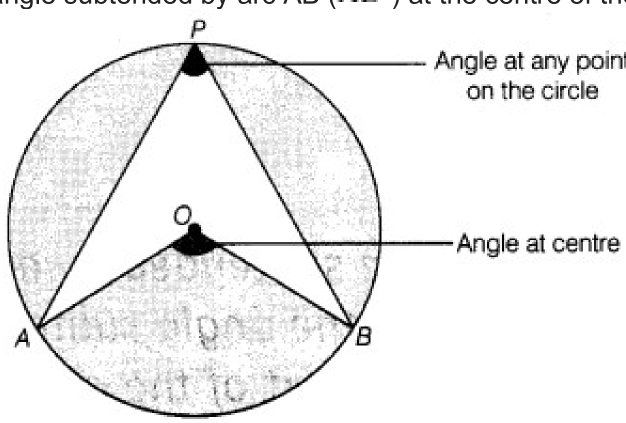


Fig. 23.2

   Also, ∠APS is the angle subtended by arc AB ($\bar{AB}$) at a point P on the remaining part of the
   circle.
7. Important points about angle subtended by an arc

1. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
2. Angles in the same segment of a circle are equal.
3. Angle in a semi-circle is a right angle.
4. If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e. they are concyclic).

Procedure

1. Take a rectangular piece of cardboard of suitable size and by using adhesive, paste a white paper on it.
2. Cut out a circle of suitable radius with centre O from drawing sheet and paste it on the cardboard.
3. Take a pair of points O and R on the circle to obtain the arc QR. (see Fig. 23.3)

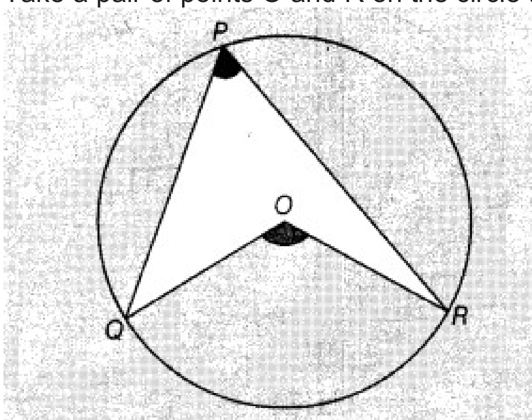


Fig. 23.3

4. To obtain the angle subtended by arc QR at centre O, join the points O and R to the centre O. (see Fig. 23.3)
5. Taking a point P on the remaining part of circle, join it to Q and R to get ∠QPR subtended by arc QR on point P on the remaining part of circle, (see Fig. 23.3)
6. Mark ∠QPR and ∠QOR.
7. Make a cut out of ∠QOR and a pair of cut outs of ∠QPR using transparent sheet, (see Fig. 23.4)

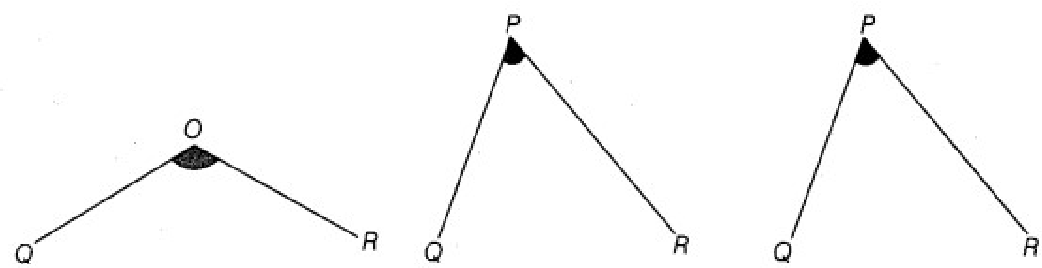


Fig. 23.4

8. Now, place the pair of cut outs of ∠QPR on the cut out of ∠QOR, adjacent to each other, (see Fig. 23.5)

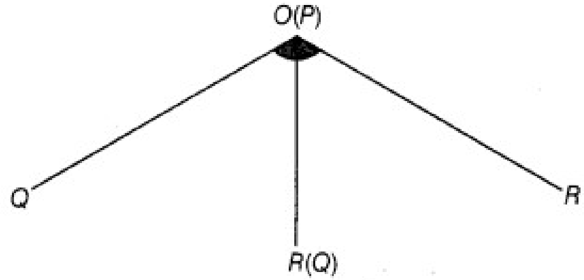


Fig. 23.5

Demonstration

Here, ∠QOR = 2 ∠QPR

We find that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of circle.

Observation

By actual measurement, ∠QOR = ………… ,

∠QPR = ………… ,

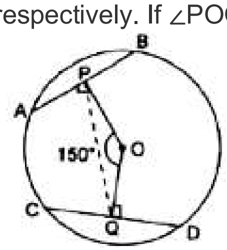
Therefore, ∠QOR = 2 ………… .

Result

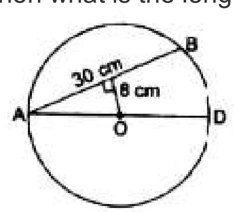
We find that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Practice Questions

1. In the figure, AB and CD are two equal chords of the circle with centre. O. OP and OQ are perpendiculars on chords AB and CD respectively. If ∠POQ = 150^0, then what is ∠APQ?



2. AD is a diameter of a circle and AB is a chord. If AB = 30 cm and its perpendicular distance from the centre of the circle is 8 cm, then what is the length of the diameter AD?



3. A circle of 30 cm diameter has a 24 cm chord What is the distance of the chord from the centre?

4. A chord AB of a circle with centre O is 10 cm. If the chord is 12 cm away f om centre, then what is the radius of the circle?

5. If the diameter AD of a circle is 34 cm and the length of a chord AB is 30 cm. What is the distance of AB from the centre?

6. What is the length of a chord which is at a distance of 4 cm from the centre of a circle of radius 5 cm?

7. If the radius of a circle is 13 cm and the length of its chord is 10 cm then what is the distance of chord from the centre?

8. If the distance of 10 cm long chord from the centre of the circle is 12 cm then what is the diameter of the circle?