BAL BHARATI PUBLIC SCHOOL, PITAMPURA, DELHI – 110034

SUBJECT:- MATHEMATICS

Class IX

CHAPTER10:- CIRCLES

Week : 5th October to 9th October'2020

Number of blocks : 4

Subtopics :

- Introduction
- Circles and related terms
- Equal chords
- Distance of equal chords from the centre
- Perpendicular from the centre to a chord

Link for the chapter : http://ncert.nic.in/textbook/textbook.htm?jemh1=3-15

Learning Outcomes:

Each student will be able to :

- Define Circle
- Define different terms related to a circle and draw their figures
- Prove that equal chords subtend equal angles at the centre and its converse
- Prove that perpendicular from the centre bisects the chord and its converse

Teaching Aids Used :

Presentation of E-lesson, PDF of NCERT textbook, YouTube videos by screen sharing, white board and marker or register and pen using laptop/mobile camera, digital board, Google Jamboard etc.

GUIDELINES:

Dear Students
Kindly read the content given below and view the links shared for better understanding.

Solve the given questions in yellow register provided in the notebook set.

Link for the chapter : http://ncert.nic.in/textbook/textbook.htm?jemh1=3-15

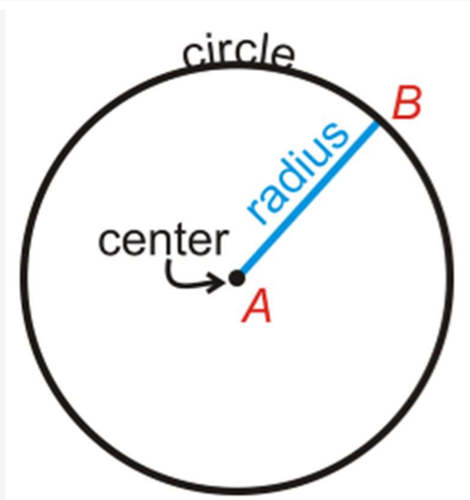
DAY 1

INTRODUCTION TO CIRCLES:

There are lot many objects in our life which are round in shape. Few examples are the clock, dart board, cartwheel, ring, vehicle wheel, Coins, etc.

Circles

- Any closed shape in a plane with all points connected at equidistance from a point in the same plane forms a Circle.
- Any point in the plane of a circle which is at equidistance from anywhere from its boundary is known as the **Centre of the Circle**.
- Radius is a Latin word which means 'ray' but in the circle it is the line segment from the centre of the Circle to its edge. So, any line starting or ending at the centre of the circle and joining anywhere on the border on the circle is known as the **Radius of Circle**.

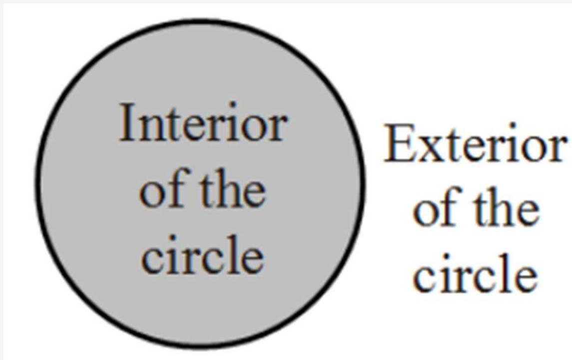


LESSON DEVELOPMENT

Interior and Exterior of a Circle

In a flat surface, the **interior of a circle** is the line whose distance from the centre is less than the radius.

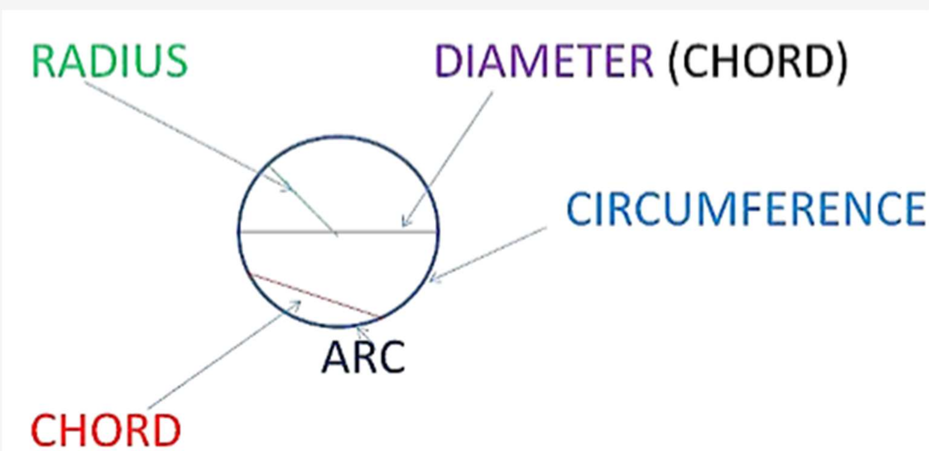
The **exterior of a circle** is the line in the plane whose distance from the centre is larger than the r adius.



We can say that

- The **set of all the points** in a plane that is at a **fixed distance** from a **fixed point** makes a circle.
- The **Fixed point** from which the set of points are at fixed distance is called the **centre** of the circle.
- A circle divides the plane into 3 parts: **interior** (inside the circle), the **circle** itself and **exterior** (outside the circle).

Terms related to circle



Radius

– The **distance** between the **centre** of the circle and any **point on its edge** is called the **radius.**

Chord

-The **line segment** within the circle joining any 2 points on the circle is called the chord.

Diameter

– A **Chord** passing through the centre of the circle is called the **diameter.** – The **Diameter is 2 times the radius** and it is the **longest chord**.

Arc

– The **portion** of a circle(curve) **between 2 points** is called an **arc**. – Among the two pieces made by an arc, the **longer** one is called a **major arc** and the **shorter** one is called a **minor arc.**

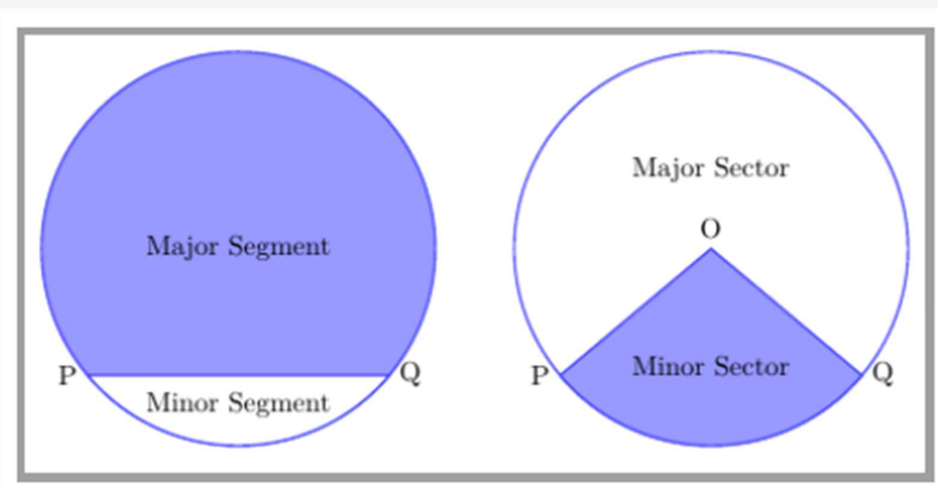
Circumference

The **perimeter** of a circle is the **distance** covered by going around its **boundary once**. The perimeter of a circle has a special name: **Circumference**, which is π times the diameter which is given by the formula 2πr

Segment and Sector

– A circular **segment** is a region of a circle which is "**cut off**" from the rest of the circle by a secant or a chord. – **Smaller region** cut off by a chord is called **minor segment** and the **bigger region** is called **major segment**. –

-A **sector** is the portion of a circle **enclosed by two radii and an arc**, where the **smaller area** is known as the **minor sector** and the **larger** being the **major sector**.



– For **2 equal** arcs or for semicircles – both the segment and sector is called the **semicircular region.**

Ex 10.1

Question 2.
Write True or False. Give reason for your answers.
(i) Line segment joining the centre to any point on the circle is a radius of the circle.
(ii) A circle has only finite number of equal chords.
(iii) If a circle is divided into three equal arcs, each is a major arc.
(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.
(v) Sector is the region between the chord and its corresponding arc.
(vi) A circle is a plane figure.

ASSIGNMENT:

Solve the following question from NCERT book in yellow register
1.Exercise 10.1 question number 1.
2. Revise all the terms related to a circle.

Links for reference :

https://youtu.be/QJoHC60Hcgs

https://youtu.be/arW0RbaBnLE

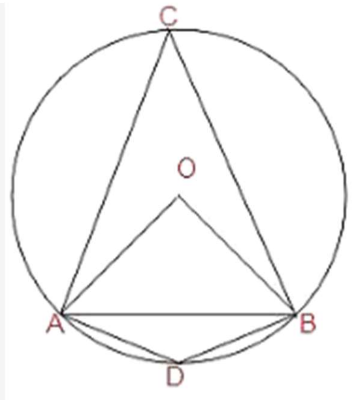
DAY 2

LESSON DEVELOPMENT

Circles and Their Chords

Angle Subtended by a Chord at a Point

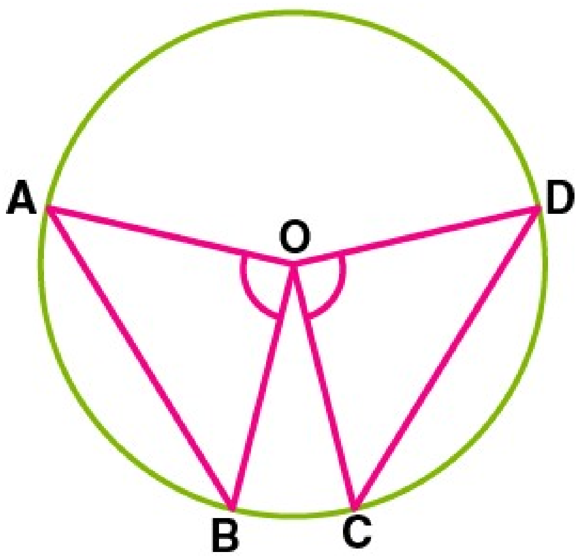
If in a circle AB is the chord and is making ∠ACB at any point of the circle, then this is the angle s ubtended by the chord AB at the point C.



Likewise, ∠AOB is the angle subtended by the chord AB at point O i.e. at the centre and ∠ADB is also the angle subtended by AB at point D on the circle.

Theorem of equal chords subtending angles at the centre.

Theorem 10.1 – Equal **chords of a circle** subtend equal **angles at the centre**.



Proof: AB and CD are the 2 equal chords.

In Δ AOB and Δ COD

OB = OC [Radii]

OA = OD [Radii]

AB = CD [Given]

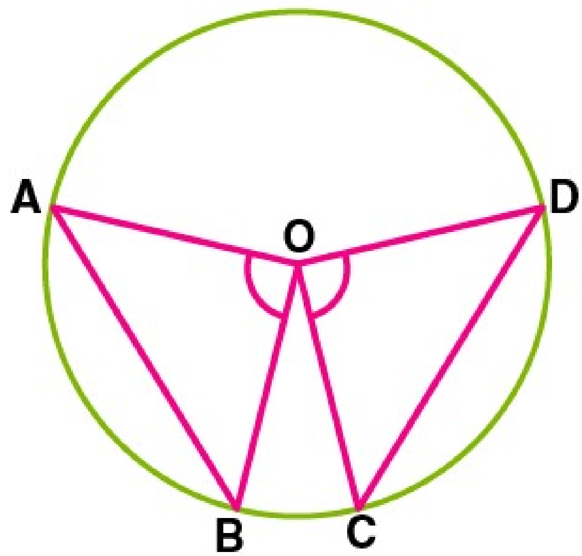
ΔAOB ≅ ΔCOD (SSS rule)

Hence, ∠AOB = ∠COD [CPCT]

Theorem of equal angles subtended by different chords.

Theorem 10.2 – If the **angles** subtended by the chords of a circle at the centre are **equal**, then the **chords are equal.**

Proof: In ΔAOB and ΔCOD



OB = OC [Radii]

∠AOB=∠COD [Given]

OA = OD [Radii]

ΔAOB ≅ ΔCOD (SAS rule)

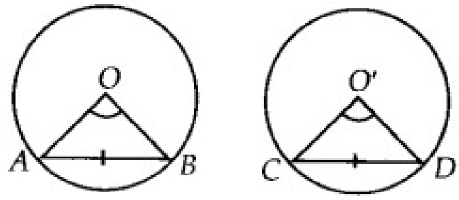
Hence, AB=CD [CPCT]

Ex 10.2

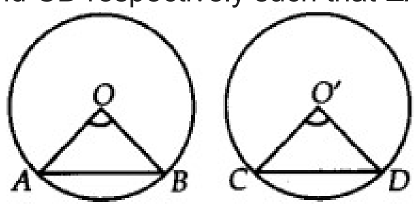
Question 1.
Recall that two circles are congruent, if they have the same radii.

Prove that equal chords of congruent circles subtend equal angles at their centres
Solution:
Given: Two congruent circles with centres O and O' and radii r, which have chords AB and CD respectively such that AB = CD.
To Prove: ∠AOB = ∠CO'D
Proof: In ΔAOB and ΔCO'D, we have
AB = CD [Given]
OA = O'C [Each equal to r]
OB = O'D [Each equal to r]
∴ ΔAOB ≅ ΔCO'D [By SSS congruence criteria]

⇒ ∠AOB = ∠CO’D [C.P.C.T.]



Question 2.
Prove that, if chords of congruent circles subtend equal angles at their centres, then the chords are equal.
Solution:
Given: Two congruent circles with centres O & O’ and radii r which have chords AB and CD respectively such that ∠AOB = ∠CO’D.



To Prove: AB = CD
Proof: In ∆AOB and ∆CO’D, we have
OA = O’C [Each equal to r]
OB = O’D [Each equal to r]
∠AOB = ∠CO’D [Given]
∴ ∆AOB ≅ ∆CO’D [By SAS congruence criteria]
Hence, AB = CD [C.P.C.T.]

Link for reference :

https://youtu.be/pYvaf5h3fSo

ASSIGNMENT

1.Learn the theorems and understand their applications.

MCQs for practice

1) The center of the circle lies in______ of the circle.

a. Interior b. Exterior c. Circumference d. None of the above

2) The longest chord of the circle is:

a. Radius b. Arc c. Diameter d. Segment

3) Equal _____ of the congruent circles subtend equal angles at the centers.

a. Segments b. Radii c. Arcs d. Chords

4) If chords AB and CD of congruent circles subtend equal angles at their centres, then:

a. AB = CD b. AB > CD

c. AB < AD d . None of the above

5) If there are two separate circles drawn apart from each other, then the maximum number of common points they have:

a. 0 b. 1 c. 2 d. 3

Links for reference :

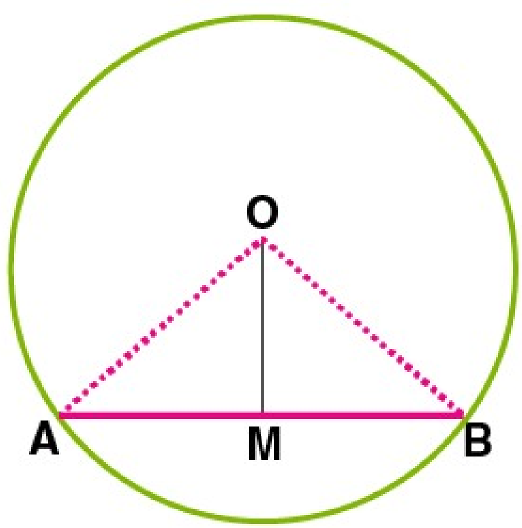
1. https://youtu.be/lmn-kNEA-JQ

DAY 3

LESSON DEVELOPMENT

Perpendicular from the centre to a chord

Theorem 10.3 : Perpendicular from the **centre** of a circle to a **chord bisects the chord**.



Proof: AB is a chord and OM is the perpendicular drawn from the centre.

From ΔOMB and ΔOMA,

$\angle OMA = \angle OMB = 90^0$ OA = OB (radii)

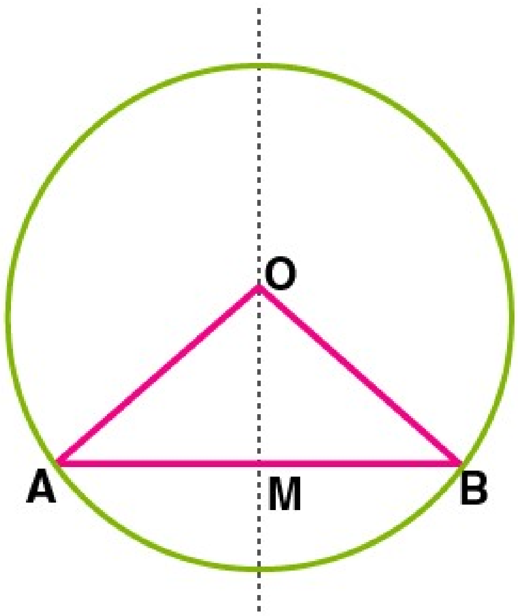
OM = OM (common)

Hence, $\Delta OMB \cong \Delta OMA$ (RHS rule)

Therefore AM = MB [CPCT]

A Line through the centre that bisects the chord is perpendicular to the chord.

Theorem 10.4 – A **line drawn** through the centre of a circle to **bisect** a chord is **perpendicular** to the chord.



Proof: OM is drawn from the center to bisect chord AB.

In ΔOMA and ΔOMB,

OA = OB (Radii)

OM = OM (common)

AM = BM (Given)

Therefore, $\Delta OMA \cong \Delta OMB$ (SSS rule)

$\Rightarrow \angle OMA = \angle OMB$ (C.P.C.T)

But, $\angle OMA + \angle OMB = 180^0$

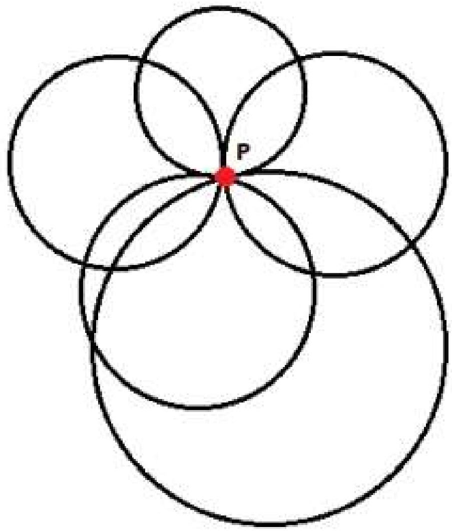
Hence, $\angle OMA = \angle OMB = 90^0 \Rightarrow OM \perp AB$

RELATED LINK :

https://youtu.be/xL7OPb9uIOc

Circle Passing Through a Point

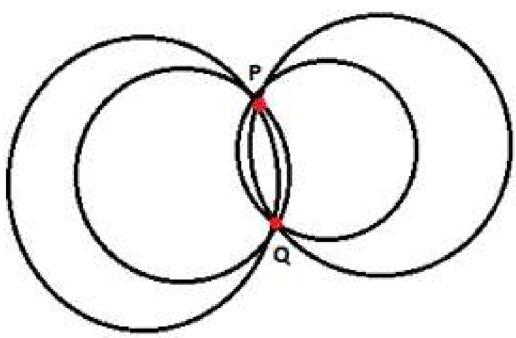
Let us consider a point and try to draw circle passing through that point.



It can be seen that through a single point P ,infinite circles passing through it can be drawn.

Circle Passing Through Two Points

Now, let us take two points P and Q and see what happens.



Again ,we see that an infinite number of circles passing through points P and Q can be drawn.

Circle Passing Through Three Points (Collinear or Non-Collinear)

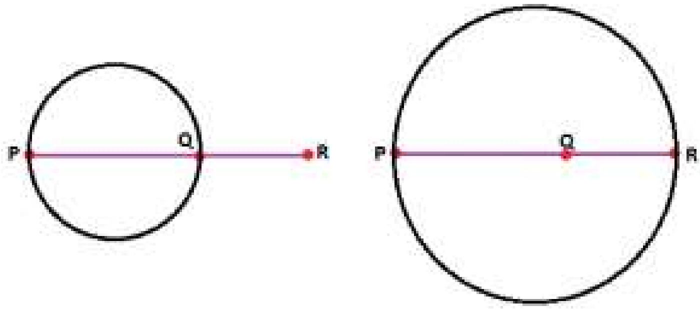
Let us now take 3 points. For a circle passing through 3 points, two cases can arise.

- Three points can be collinear
- Three points can be non-collinear

Let us study both cases individually.

Case 1: A circle passing through 3 points: Points are collinear

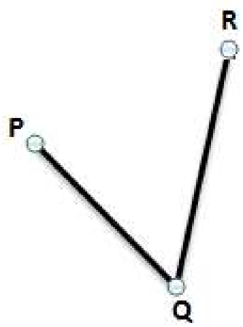
Consider three points P, Q and R which are collinear.



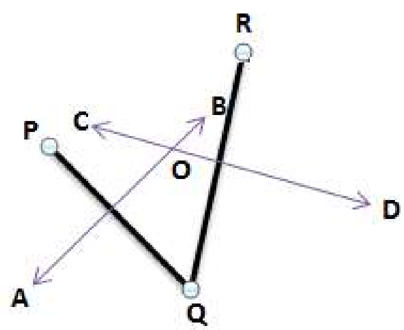
It can be seen that if three points are collinear any one of the points either lie outside the circle or inside it. Therefore, a circle passing through 3 points, where the points are collinear is not possible.

Case 2: A circle passing through 3 points: Points are non-collinear

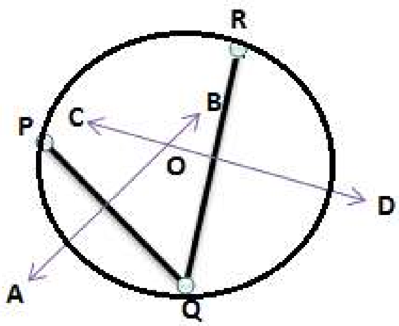
To draw a circle through three non-collinear points join the points as shown:



Draw perpendicular bisectors of PQ and RQ. Let the bisectors AB and CD meet at O.



With O as the centre and radius OP or OQ or OR draw a circle. We get a circle passing through 3 point P, Q, and R.



It is observed that only a unique circle will pass through all the three points. It can be stated as a theorem 10.5 .

Theorem 10.5 – There is one and only one circle passing through three given non collinear points .

RELATED LINK :

https://youtu.be/IKiYV_A-Jtg

ASSIGNMENT:

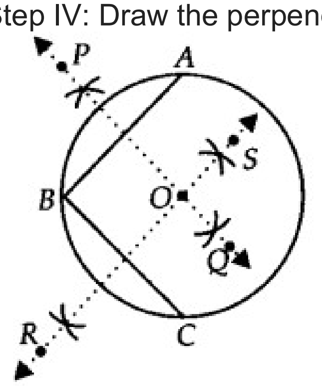
1.Learn the theorems and understand their applications.

DAY 4

LESSON DEVELOPMENT

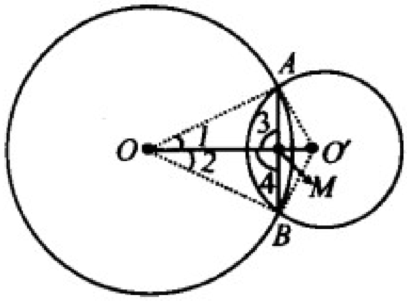
Ex 10.3

Question 2.
Suppose you are given a circle. Give a construction to find its centre.
Solution:
Steps of construction :
Step I : Take any three points on the given circle. Let these points be A, B and C.
Step II : Join AB and BC.
Step III : Draw the perpendicular bisector, PQ of AB.
Step IV: Draw the perpendicular bisector, RS of BC such that it intersects PQ at O.



Thus, ‘O’ is the required centre of the given drcle.

Question 3.
If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.
Solution:
We have two circles with centres O and O', intersecting at A and B.
∴ AB is the common chord of two circles and OO' is the line segment joining their centres.
Let OO' and AB intersect each other at M.



∴ To prove that OO' is the perpendicular bisector of AB,
we join OA, OB, O'A and O'B. Now, in ΔQAO' and ΔOBO',
we have
OA = OB [Radii of the same circle]
O'A = O'B [Radii of the same circle]
OO' = OO' [Common]
∴ ΔOAO' ≅ ΔOBO' [By SSS congruence criteria]
⇒ ∠1 = ∠2 , [C.P.C.T.]
Now, in ΔAOM and ΔBOM, we have
OA = OB [Radii of the same circle]
OM = OM [Common]
∠1 = ∠2 [Proved above]
∴ ΔAOM = ΔBOM [By SAS congruence criteria]
⇒ ∠3 = ∠4 [C.P.C.T.]
But ∠3 + ∠4 = 180° [Linear pair]
∴∠3=∠4 = 90°
⇒ AM ⊥ OO'
Also, AM = BM [C.P.C.T.]
⇒ M is the mid-point of AB.
Thus, OO' is the perpendicular bisector of AB.

Write whether True or False and justify your answer

Question 1:
Two chords AB and CD of a circle are each at distances 4 cm from the centre. Then, AB = CD.

Question 2:
Two chords AB and AC of a circle with centre O are on the opposite sides of OA. Then, ∠OAB = ∠OAC.

Question 3:
The congruent circles with centres O and O’ intersect at two points A and B. Then, $\angle AOB = \angle AO'B$.

Question 4:
Through three collinear points a circle can be drawn.

Question 5:
A circle of radius 3 cm can be drawn through two points A, B such that AB = 6 cm.

ASSIGNMENT:

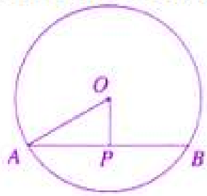
Solve the following question from NCERT book in yellow register
1.Exercise 10.3 question number 1.

Practice questions

Extra Questions for Class 9 Maths Chapter 10 Circles

Question-1

The radius of a circle is 13 cm and length of one of its chords is 24 cm. Find the distance of the chord from the centre.



Solution:

Let AB = 24 cm be the chord of the circle with radius AO = 13 cm.

Draw OP $\perp$ AB. Join OA according to theorem,

AP= $\frac{1}{2}AB = \frac{1}{2}\times 24 = 12$ cm

In Δ APO, $\angle P = 90°$

$\therefore AO^2 = AP^2 + OP^2$

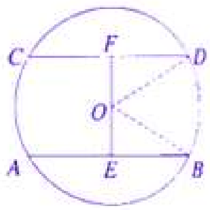
$= 12^2 + OP^2$

$\Rightarrow 13^2 = 12^2 + OP^2$

$\Rightarrow OP = 5$ cm

Question-2

AB and CD are two parallel chords of a circle such that AB = 10 cm and CD = 24 cm. If chords are on opposite sides of the centre of the circle and distance between them is 17 cm find the radius of the circle.



Solution:

Let AB and CD be two parallel chords of the circle C(O, r). Draw OE $\perp$ AB and OF $\perp$ CD since AB || CD, hence points E, O and F will be collinear, and EF = 17 cm.

Let OE = x cm, then OF = (17 - x) cm. Join 0B and OD.

This follows OB = OD = r

Again EB = $\frac{1}{2}$ AB = 5 cm

and FD = $\frac{1}{2}$CD = 12 cm

In ΔOEB, $\angle$E = 90°

$\therefore OB^2 = OE^2 + EB^2 = x^2 + 25$ .... (i) and $OD^2 = OF^2 + FD^2 = (17 - x)^2 + 144$ .... (ii)

But $OB = OD \Rightarrow OB^2 = OD^2$

From Equations (i) and (ii) we get,

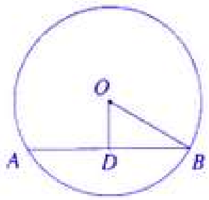
$x^2 + 25 = (17 - x)^2 + 144 \Rightarrow x = 12$

Substituting x = 12 in Equation (i).

We get $r^2 = 169 \Rightarrow r = 13$ cm
$\therefore$ The radius of the circle = 13 cm

Question-3

Determine the length of a chord, which is at a distance of 5 cm from the centre of the circle of radius 13 cm.



Solution:

Let AB be the chord of a circle of radius 13 cm.

Draw $OD \perp AB$, then $OD = 5$ cm

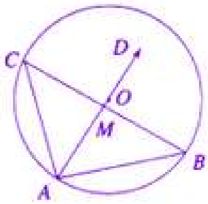
In ΔODB, $\angle D = 90°$

\ $OB^2 = OD^2 + DB^2$
$\Rightarrow 13^2 = 5^2 + DB^2 \Rightarrow DB = 12$ cm

Hence, $AB = 2 \times DB = 24$ cm.

Question-4

In the Fig. AB and AC are two equal chords of a circle. Prove that bisector of ∠BAC passes through the centre of the circle.



Solution:

Let AB and AC be two equal chords of a circle and ∠BAD = ∠CAD in ΔAMB and ΔAMC, AB = AC (given), AM = AM (Common) and
∠BAD = ∠CAD.
∴ ΔAMB @ DAMC
∴ BM = CM and ∠CMA = ∠BMA=90°
Hence, AD is a perpendicular bisector of chord BC. But perpendicular bisector of a chord passes through the centre.
Hence AD passes through the centre O.

Thus, AD = BE = CF

$\Rightarrow \frac{2}{3}AD = \frac{2}{3}BE = \frac{2}{3}CF$

⇒ GA = GB = GC (∵ the centroid divides each median in the ratio 2 : 1)

Also OA = OB = OC, where O is the circum-centre.

Hence, G coincides with O.

Using eqn (i) in eqn(ii) we get,
$36 = 25 - y^2 + (5-y)^2 \Rightarrow y = \frac{7}{5}$cm.
Putting $y = \frac{7}{5}$ cm in Eq. (i)
$25 = x^2 + \frac{49}{25}$
⇒ x = 4.8 cm
BC= 2x = 2 * 4.8 = 9.6 cm