BAL BHARATI PUBLIC SCHOOL, PITAMPURA, DELHI – 110034

SUBJECT: MATHEMATICS

CHAPTER: 1

TOPIC: Real Numbers

STEP 1: GUIDELINES AND INTRODUCTION

Guidelines:

Dear students, kindly refer to the following notes/video links from the Chapter- "Real Numbers" and thereafter do the questions in your Math register.

(Chapter1)

LINK FOR THE CHAPTER:
http://ncert.nic.in/textbook/textbook.htm?jemh1=1-15

INTRODUCTION

Real Numbers- Collection of all rational and irrational numbers form a set of real numbers. All real numbers can be plotted on a number line or real number line. Between any two rational numbers, there are infinitely many rational and irrational numbers.

STEP 2: Subtopics:

(i) Euclid's division lemma
(ii) Euclid's division algorithm
(iii) HCF and LCM of two or more numbers using prime factorization and division method
(iv) Fundamental theorem of Arithmetic
(v) Revisiting irrational numbers - Prove $\sqrt{2}$ and $\sqrt{5}$ etc. irrational.
(vi) Decimal expansion of rational and irrational numbers

STEP 3 - Key Points and Important Links to Remember:

1. Euclid's Division Lemma

Euclid's Division Lemma states that given two integers *a* and *b*, there exists a unique pair of integers *q* and *r* such that $a=b\times q+r$ and $0\le r<b$.
or *dividend = divisor × quotient + remainder*

2. Euclid's Division Algorithm

- Euclid's Division Algorithm is a method used to find the **H.C.F** of two numbers, say *a* and *b* where $a> b$
- We apply Euclid's Division Lemma to find two integers *q* and *r* such that $a=b\times q+r$ and $0\le r<b$

- If r = 0, then H.C.F is b, else, we apply Euclid's division Lemma to b (the divisor) and r (the remainder) to get another pair of quotient and remainder.
- The above method is repeated until a remainder of zero is obtained. The divisor in that step is the H.C.F of the given set of numbers.

Refer to the following link to visualize division algorithm:

https://www.khanacademy.org/math/in-in-grade-10-ncert/x573d8ce20721c073:real-numbers/x573d8ce20721c073:euclid-s-division-algorithm/v/euclids-division-algorithm-visualised

Q- Find HCF of 126, 567 and 441 using Euclid's division algorithm.

Solution. First, we find HCF of 441 and 567.

Using Euclid's division algorithm, we have

$$567 = 441 \times 1 + 126$$

$$441 = 126 \times 3 + 63$$

$$126 = 63 \times 2 + 0.$$

Thus, last non-zero remainder = 63

$\therefore$ HCF (441, 567) = 63.

Now, we find HCF of 63 and 693.

By using Euclid's division algorithm, we have

$$693 = 63 \times 11 + 0.$$

So, last non-zero remainder = 63.

$\therefore$ HCF (63, 693) = 63.

Hence, HCF of 441, 567, 693 = 63.

3. Fundamental Theorem of Arithmetic-The Fundamental Theorem of Arithmetic states that every composite number can be expressed as a product of its prime factors and this factorization is unique irrespective of the orders of prime factors.

Refer to the following link:

https://www.khanacademy.org/computing/computer-science/cryptography/modern-crypt/v/the-fundamental-theorem-of-arithmetic-1

HCF of two or more numbers using Prime Factorization Method-
https://mail.google.com/mail/u/0/?tab=wm#inbox/QgrcJHrnqzTFhgRxHndHgcrRXlTDdcclhmg?projector=1

HCF using long division method-
https://www.youtube.com/watch?v=eljVa2KqOTo&feature=youtu.be

LCM using division method

https://mail.google.com/mail/u/0/#inbox/QgrcJHrnqzTFhgRxHndHgcrRXlTDdcclhmg?projector=1

Real life application of HCF and LCM

https://mail.google.com/mail/u/0/#inbox/QgrcJHrnqzTFhgRxHndHgcrRXlTDdcclhmg?projector=1

Note- Product of Two Numbers = HCF X LCM of the two numbers

- For any **two** positive integers a and b,
  $a \times b = H.C.F \times L.C.M.$
- Example – For 36 and 56, the H.C.F is 4 and the L.C.M is 504
  $36 \times 56 = 2016$
  $4 \times 504 = 2016$
  Thus, $36 \times 56 = 4 \times 504$
- The above relationship, however, doesn't hold true for 3 or more numbers

4. Revisiting Irrational Numbers

Irrational Numbers

Any number that cannot be expressed in the form of p/q (where p and q are integers and $q \neq 0$.) is an irrational number. Examples $\sqrt{2}, \pi$ and so on.

Interesting Results

- If a number p (a prime number) divides a^2, then p divides a. Example: 3 divides 6^2 i.e. 36, which implies that 3 divides 6.
- We can prove $\sqrt{2}$, $\sqrt{5}$ irrational using method of contradiction.
- https://www.youtube.com/watch?v=IzkCVzzHHbg
- https://www.youtube.com/watch?v=m8eH4iAI-2Q (Refer to these links to prove $\sqrt{2}$ and $\sqrt{5}$ irrational and few important questions)
- **Prove that $\sqrt{7}$ is irrational**
  Assumption: $\sqrt{7}$ is rational

Since it is rational √7 can be expressed as a/b
where a and b are co-prime Integers, b ≠ 0.
On squaring, $a^2/b^2=7$
$\Rightarrow a^2=7b^2$.

- Thus 7 divides a^2
  Hence, 7 divides a. Then, there exists a number c such that a=7c.

  Then, $a^2=49c^2$. Hence, $7b^2=49c^2$ or $b^2=7c^2$.
  Hence 7 divides b.

  Since 7 is a common factor for both a and b, it contradicts our assumption that a and b are coprime integers.
  Hence, our initial assumption that √7 is rational is wrong. Therefore, √7 is irrational.

5. Decimal Expansion of Rational and Irrational numbers

To Check if a given rational number is terminating or not

If a/b is a rational number, then its decimal expansion would terminate if **both** of the following conditions are satisfied:
a) The H.C.F of a and b is 1.
b) b can be expressed as a prime factorisation of 2 and 5 i.e. $b=2^m \times 5^n$ where either m or n, or both m and n are 0.
If the prime factorisation of b contains any number other than 2 or 5, then the decimal expansion of that number will be non-terminating but recurring.

Refer to the following link for more practice:

https://www.youtube.com/watch?v=3_FTJEI-xvs

STEP 4

Points to Remember

1) Numbers whose decimal expansion is non-terminating and non-recurring are irrational numbers.
2) Product of three numbers may not be equal to the product of their HCF and LCM.
3) Every rational number has either terminating or non-terminating decimal expansion.

ASSIGNMENT

Do NCERT Ex 1.1,1.2,1.3 and 1.4 in the CW/HW register.

Do the following questions in practice notebook:
1. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$: x,y are prime numbers, then HCF(a,b)is
a) xy   b) xy^2 c) x^3y^3 d) x^2y^2
2. If two positive integers a and b are expressible in the form $a = pq^2$ and $b = p^3q$; p and q being prime numbers, LCM (a, b) =
a) pq   b) p^3q^3 c) p^3q^2 d) p^2q^2
3. If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a =

a) 2 b) 3 c)4 d)1

4. If the sum of LCM and HCF of two numbers is 1260 and their LCM is 900 more than their HCF, the product of two numbers is

a) 203400 b) 194400 c) 198400 d) 205400

5. The decimal expansion of the rational number $\frac{33}{220}$ will terminate after ________ decimal places.

NOTE

1. Refer to the following links to practice more questions:

a) https://diksha.gov.in/play/collection/do_3129243959686676481258?referrer=utm_source%3Ddiksha_mobile%26utm_content%3Ddo_3129243959686676481258%26utm_campaign%3Dshare_content

b) From Khan Academy Assignments

https://www.khanacademy.org/math/in-in-grade-10-ncert

c) www.examfear.com

d) http://www.ei-india.com/mindspark-math (free trial for 60 days)