3 Tests (3 T's) of solving higher order derivative questions.

Given, $y = \text{-----}$

find $dy/dx = \text{----}$

(*) **3 T's**

(1) Is there any term in R.H.S. denominator, if Yes, take it to L.H.S.

2) Is there any term in square root, if Yes, squaring both the sides.

3) Can we search y in $y_1 \left(\frac{dy}{dx}\right)$ if Yes, replace it.

Then, go for 2^{nd} derivative.

(*) we can solve > 90% questions through this.

Q If $y = e^{a \cos^{-1} x}$; $|x| \le 1$,

then show that

$(1-x^2) y_2 - x y_1 - a^2 y = 0$.

Here, $y = e^{a \cos^{-1} x}$.

Keep 3 T's in mind.

$\Rightarrow \frac{dy}{dx} = \underbrace{e^{a \cos^{-1} x}}_{y} \cdot \left(\frac{-a}{\sqrt{1-x^2}} \right)$.

$\Rightarrow \sqrt{1-x^2}\, y_1 = -ay$

Squaring both the sides.

$\Rightarrow (1-x^2) y_1^2 = a^2 y^2$.

Diff. w.r.to x.

$\Rightarrow (1-x^2) \cdot 2 y_1 \cdot y_2 + y_1^2 (-2x) = 2a^2 y y_1$

Cancelling $2y_1$

$\Rightarrow (1-x^2) y_2 - x y_1 - a^2 y = 0$

Q If $y = \left[\log\left(x+\sqrt{x^2+1}\right)\right]^2$

P.T. $xy_1 + y_2(1+x^2) - 2 = 0$

Here, $y = \left[\log\left(x+\sqrt{x^2+1}\right)\right]^2$

$$\Rightarrow \frac{dy}{dx} = 2\cdot\frac{\log\left(x+\sqrt{x^2+1}\right)}{\left(x+\sqrt{x^2+1}\right)}\left(1+\frac{1\cdot \cancel{2}x}{\cancel{2}\sqrt{x^2+1}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\overbrace{\log\left(x+\sqrt{x^2+1}\right)}^{\sqrt{y}}}{\left(\cancel{x+\sqrt{x^2+1}}\right)}\cdot\left(\frac{\cancel{x+\sqrt{x^2+1}}}{\sqrt{x^2+1}}\right)$$

$$\Rightarrow y_1 = \frac{2\sqrt{y}}{\sqrt{1+x^2}}$$

{ Taking R.H.S. or in L.H.S & sq.

$$\Rightarrow (1+x^2)\,y_1^2 = 4y.$$

Diff. w.r.t. x

$$\Rightarrow (1+x^2)\cdot 2y_1\cdot y_2 + y_1^2\cdot 2x = 4y_1$$

Cancelling $2y_1$

$$\Rightarrow (1+x^2)\,y_2 + xy_1 - 2 = 0.$$

Try the following

1) If $y = \{x + \sqrt{x^2+1}\}^m$, p.t.

$$(1+x^2)\cdot y_2 + xy_1 - m^2 y = 0.$$

2) If $x = \sin\left(\frac{1}{a}\log y\right)$

Prove that

$$(1-x^2)\, y_2 - xy_1 - a^2 y = 0$$

3) If $y = x^x$, p.t.

$$y_2 - \frac{y_1^2}{y} = y/x$$

4) If $x^m y^n = (x+y)^{m+n}$, p.t.

$$y_2 = 0$$

5) If $e^y(1+x) = 1$, p.t.

$$y_2 = y_1^2.$$