# Differentiability

A function $y = f(x)$ is said to be differentiable at $x = a$, if

$$L.H.D = R.H.D.$$

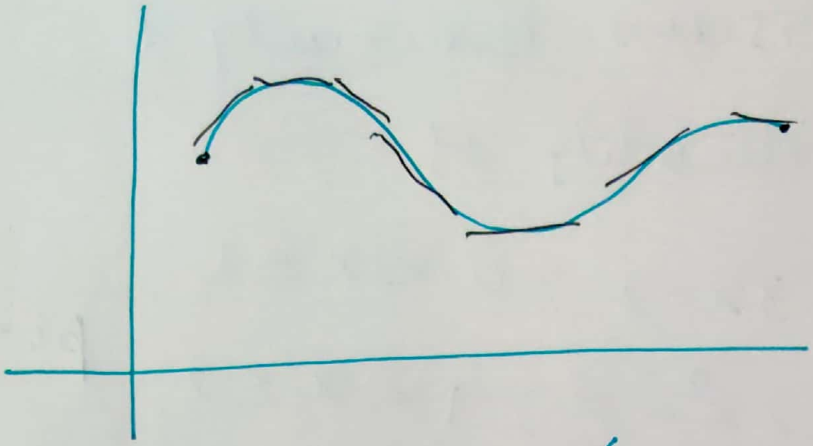
or, $Lf'(a) = Rf'(a)$.

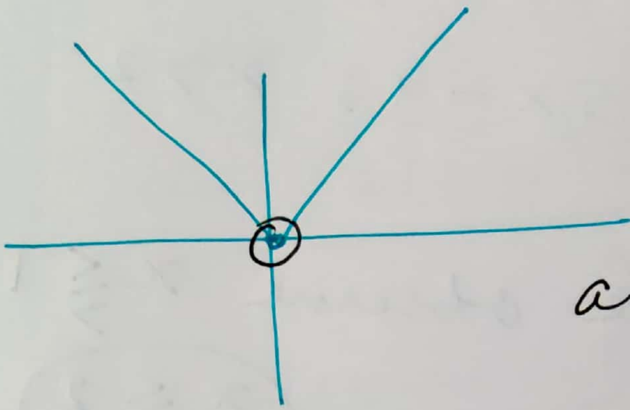
Here, $$Lf'(a) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}.$$

$$Rf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

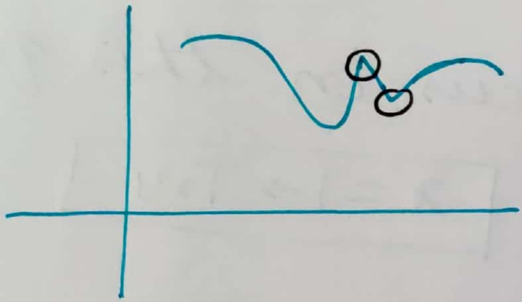
**Recall** (Graphical Meaning)
Meaning of derivative

A function $y = f(x)$ is said to be differentiable if we can draw tangents at every point on the curve.





we cannot draw tangents at the corner.



we observe. curve is continuous but not differentiable.

Note:- A function which is continuous is not ~~necessa~~ necessarily ~~continuous~~ differentiable,

But a differentiable function is always continuous.

Ex:- Discuss continuity & differentiability of

$$f(x) = \begin{cases} 3x-2 ; & 0 < x \le 1 \\ 2x^2 - x ; & 1 < x \le 2 \\ 5x-4 ; & x > 2 . \end{cases}$$

pt → ?

Here, we observe $x \lessgtr 1$ &

$x \lessgtr 2$

we shall discuss in this question

at $x = 2$, $\boxed{x = 1 \to \text{Try}.}$

L.H.L. $x = 2-h$, when $x \to 2 \Rightarrow h \to 0$

$\therefore \lim\limits_{h \to 0} 2(2-h)^2 - (2-h).$

$= \lim\limits_{h \to 0} 2(4-4h+h^2) - 2 + h . = 6$

R.H.L. $x = 2+h$, when $x \to 2 \Rightarrow h \to 0$

$\therefore \lim\limits_{h \to 0} 5(2+h) - 4 = 6$

Also, $f(2) = 6$ Hence, contin ✓

We know.

$$Lf'(a) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}.$$

$$Rf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

Since, pt is $x=2$

$$\therefore Lf'(2) = \lim_{h \to 0} \frac{f(2-h) - f(2)}{-h}.$$

$$= \lim_{h \to 0} \frac{(2(2-h)^2 - (2-h)) - 6}{-h}$$

$$= \lim_{h \to 0} \frac{2(4-4h+h^2) - 2 + h - 6}{-h}.$$

$$= \lim_{h \to 0} \frac{\cancel{8} - 8h + h^2 - \cancel{8} + h}{-h}$$

$$= \lim_{h \to 0} \frac{h^2 - 7h}{-h} = \lim_{h \to 0} \frac{\cancel{h}(h-7)}{-\cancel{h}}$$

$$= 7.$$

$$Rf'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}.$$

$$= \lim_{h \to 0} \frac{(5(2+h) - 4) - 6}{h} = \lim_{h \to 0} \frac{5\cancel{h}}{\cancel{h}} = 5$$

Here L.H.D $\neq$ R.H.D.

$\Rightarrow$ f(x) is continuous but not differentiable.

## Try the following

Examine the continuity & differentiability of f(x) ~~at x=0~~

1) $f(x) = \begin{cases} 1-x^2 ; & x \leq 0 \\ 1+x^2 ; & x > 0 \end{cases}$ at $x=0$

2). $f(x) = |x-3|$ at $x=3$.

3) $f(x) = x|x|$ at $x=0$.

(*) 4) $f(x) = [x]$ at $x=2$

** 5) If the function f defined by

$$f(x) = \begin{cases} x^2+3x+a ; & x \leq 1 \\ bx+2 ; & x > 1 \end{cases}$$

is differentiable, then find the values of a & b.

6) ~~C & D of~~ Differentiability only

$$f(x) = \begin{cases} |2x-3|\,[x] & ; x \geq 1 \\ \sin \frac{\pi x}{2} & ; x < 1 \end{cases}$$

7) $$f(x) = \begin{cases} |x-2| + 2 & ; x \leq 2 \\ x^2 - 2 & ; x > 2. \end{cases}$$

8) $$f(x) = \begin{cases} x\,[x] & ; 0 \leq x < 2 \\ (x-1)\cdot x & ; 2 \leq x < 3. \end{cases}$$

9) $$f(x) = \begin{cases} x^2 \cdot \sin 1/x & ; x \neq 0 \\ 0 & ; x = 0. \end{cases}$$

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(*) Ex 5.1, Qs :- 7, 9, 18, 19, 21,

23, 24, 26 - 34.

& Solved Ex - 20.