BAL BHARATI PUBLIC SCHOOL, PITAMPURA, DELHI – 110034

SUBJECT:- MATHEMATICS

Class-IX

CHAPTER:- 2 (PART-6)

TOPIC:-POLYNOMIALS

GUIDELINES

Dear Students
Kindly read the content given below and view the links shared for better understanding.

- **Solve the given NCERT questions in the yellow register provided in the notebook set.**

Link for the chapter : http://ncert.nic.in/textbook/textbook.htm?jemh1=3-15

Introduction :

Let us Recall the following concepts:

The Remainder Theorem

When we divide $p(x)$ by the simple polynomial $x-a$ we get:

$$p(x) = (x-a)\cdot q(x) + r(x)$$

$x-a$ is **degree 1**, so $r(x)$ must have **degree 0**, so it is just some constant r:

$$p(x) = (x-a)\cdot q(x) + r$$

Now see what happens when we have x equal to a:

$$p(a) = (a-a)\cdot q(a) + r$$

$$p(a) = (0)\cdot q(a) + r$$

$$p(a) = r$$

So we get this:

The Remainder Theorem:

When we divide a polynomial p(x) by **x-a** then the remainder is **p(a).**

So, to find the remainder after dividing by **x-a,** we need not do any division:

Just calculate p(a).

Let us see an example:

Example: The remainder after $2x^2-5x-1$ is divided by $x-3$

(Our example from above)

We don't need to divide by **(x−3)** ... just calculate **f(3)**:

$$2(3)^2-5(3)-1 = 2x9-5x3-1$$
$$= 18-15-1$$
$$= \mathbf{2}$$

And that is the remainder we got from our calculations above.

We didn't need to do Long Division at all!

Subtopics:

The Factor Theorem

Now ...

What if we calculate p(x) and it is a zero?

.... That means the remainder is zero, and ....

x-a must be a **factor** of the polynomial p(x).

Example: x^2-3x-4

$$f(4) = (4)^2-3(4)-4 = 16-12-4 = 0$$

so $(x-4)$ must be a factor of x^2-3x-4

Therefore according to the Factor Theorem :

If p(x) is a polynomial of **degree $n \geq 1$** and a is any real number, then

(i) x — a is a factor of p(x) if p(a) = 0, and

(ii) p(a) = 0, if x-a is a factor of p(x).

Proof by Remainder Theorem

By **Remainder Theorem**,

p(x)= (x—a) q(x) + p(a)

(i) If p(a) = 0, then p(x) = (x—a) q(x), which shows that x—a is a factor of p(x).

(ii) Since (x—a) is a factor of p(x), p(x) = (x—a) g(x) for some polynomial g(x).

In this case p(a)=(a—a) g(a) = 0

Proof of the theorem : https://youtu.be/19M-c5tMmIs

Important :-

1. If f(-c) = 0, then (x + c) is a factor of the polynomial f(x).
2. If p(d/c) = 0, then (cx—d) is a factor of the polynomial p(x).
3. If p(-d/c) = 0, then (cx+d) is a factor of the polynomial p(x).
4. If p(c) = 0 and p(d) = 0, then (x — c) and (x — d) are both factors of the polynomial p(x).

Example 1: Examine whether x + 2 is a factor of $p(x) = x^3 + 3x^2 + 5x+6$

Solution : The zero of x + 2 is –2

As, per factor theorem, x + 2 is a factor of p(x), if p(-2) = 0.

$p(-2) = (-2)3 + 3(-2)2 + 5(-2) + 6 = 0.$

Thus, $x+2$ is factor of $p(x) = x^3 + 3x^2 + 5x + 6$

Example 2: Show that $(x-1)$ is a factor of the polynomial $x^3 + 4x^2 + x - 6$.

Solution: Let $p(x) = x^3 + 4x^2 + x - 6$ be the given polynomial. By factor theorem, $(x-a)$ is a factor of a polynomial $p(x)$, if $p(a) = 0$. Therefore, in order to prove that $x - 1$ is a factor of $p(x)$, we have to show that $p(1) = 0$.

Now , $P(x) = x^3 + 4x^2 + x - 6$

$P(1) = 1^3 + 4 \times 1^2 + 1 - 6$

$= 1 + 4 + 1 - 6 = 0$

Hence, $(x-1)$ is a factor of $p(x) = x^3 + 4x^2 + x - 6$

Example 3: Without actual division, prove that $4x^4 - 2x^3 - 6x^2 + x + 1$ is exactly divisible by $2x^2 + x - 1$.

Solution: Let $f(x) = 4x^4 - 2x^3 - 6x^2 + x + 1$ and $g(x) = 2x^2 + x - 1$ be the given polynomials.

We have, $g(x) = 2x^2 + x - 1$

$= \mathbf{2x^2 + 2x - 1x - 1}$ **(Please note this step)**

$= 2x(x+1)-1(x+1) = (2x-1)(x+1)$

Therefore, $(2x-1)$ and $(x+1)$ are factors of $g(x)$

In order to prove that $f(x)$ is exactly divisible by $g(x)$, it is sufficient to prove that $2x-1$ and $x+1$ are factors of $f(x)$. For this it is sufficient to prove that $f(1/2) = 0$ and $f(-1) = 0$.

Now, $f(x) = 4x^4 - 2x^3 - 6x^2 + x + 1$

$f(1/2) = 4 . (1/2)^4 - 2. (1/2)^3 - 6 .(1/2)^2 + 1/2 + 1$

$= 1/4 - 1/4 - 3/2 + 1/2 + 1 = 0$

$f(-1) = 4 . (-1)^4 - 2 . (-1)^3 - 6 . (-1)^2 - 1 + 1$

$= 4 + 2 - 6 - 1 + 1 = 0$

(2x–1) and (x+1) are factors of f(x)

g(x) = (2x–1)(x+1) is a factors of f(x)

Hence, f(x) is exactly divisible by g(x)

Example 4: Find the value of a, if $(x + a)$ is a factor of $x^3 + ax^2 - 2x + a + 4$.

Solution: Let $p(x) = x^3 + ax^2 - 2x + a + 4$ be the given polynomial. By Factor Theorem, $(x + a)$ is a factor of p(x)

if $p(-a) = 0$.

Now, p(-a) = 0

$(-a)^3 + a(-a)^2 - 2(-a) + a + 4 = 0$

$-a^3 + a^3 + 2a + a + 4 = 0$

$3a + 4 = 0$

$a = -4/3$

Hence, (x + a) is a factor of the given polynomial, if a = – 4/3

Key points and important links for reference:

Application of Factor Theorem with an example :

https://examfear.com/free-video-lesson/Class-9/Maths/Polynomials/part-12/Polynomials_Part_12_(Factorization_using_factor_theorem_).htm

Visit https://examfear.com/ for further reference.

ASSIGNMENT :-

To be done in the yellow register.

Ex 2.4 :

Q 1. ii and iv part
Q 2. ii part
Q 3. Ii and iv part

QUESTIONS FOR PRACTICE

Note : Following questions are for practice only and should be done in a separate practice register/copy of mathematics .

Application of the Factor Theorem

Find whether $x - c$ is a factor of the polynomial f(x) or not in the following cases.

1. $f(x) = 4x^3 - 3x^2 - 8x + 4, \quad c = 3$
2. $f(x) = 3x^4 - 6x^3 - 5x + 10, \quad c = 1$
3. $f(x) = 3x^6 + 2x^3 - 176, \quad c = -2$
4. $f(x) = 4x^6 - 64x^4 - x^2 - 16, \quad c = 4$
5. $f(x) = 2x^4 - x^3 - 2x - 1, \quad c = -1/2$

2. The value of k, if $(x - 1)$ is a factor of $4x^3 + 3x^2 - 4x + k$, is

(a) 1
(b) 2
(c) -3
(d) 3