BAL BHARATI PUBLIC SCHOOL, PITAMPURA,DELHI – 110034

SUBJECT:- MATHEMATICS

CHAPTER: 4

TOPIC: Quadratic Equation

STEP 1: GUIDELINES AND INTRODUCTION

Guidelines:

Dear students, kindly refer to the following notes/video links from the Chapter- "Quadratic Equation" and thereafter do the questions in your Maths notebook.

(Chapter4 – part 2)

LINK FOR THE CHAPTER: http://ncert.nic.in/textbook/textbook.htm?jemh1=4-15

INTRODUCTION

Solution of the quadratic equation: The value of the variable x that satisfies the given quadratic equation

STEP 2: Subtopics:

(i) Solution of Quadratic equation using quadratic formula
(ii) Forming a quadratic equation in a given situation and solving it
(iii) Nature of roots of the given quadratic equation

STEP 3: Key Points and Important Link for References

(i) Quadratic Formula

For the Quadratic Equation $ax^2 + bx + c = 0$, $a \neq 0$, if $b^2 - 4ac \geq 0$, then the roots are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

when $ax^2+bx+c=0$

a, b, c = constants, where $a \neq 0$

Refer to the following link for more practice:

https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:quadratic-formula-a1/v/using-the-quadratic-formula

https://www.youtube.com/watch?v=3ayhvAI3IeY

(iii) $4x^2 - 4\sqrt{3}x + 3 = 0$

This is of the form $ax^2 + bx + c = 0$,

where $a = 4, b = 4\sqrt{3}$ and $c = 3$.

Discriminant, $D = b^2 - 4ac$

$= (4\sqrt{3})^2 - 4 \times 4 \times 3 = 48 - 48 = 0$

Since, $D = 0$

Roots are

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-4\sqrt{3} + 0}{8} = \frac{-4\sqrt{3}}{8} = \frac{-\sqrt{3}}{2}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-4\sqrt{3} - 0}{8} = \frac{-4\sqrt{3}}{8} = \frac{-\sqrt{3}}{2}$$

Hence, the roots are $\frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$.

Equation Reducible to Quadratic Equations

Given: $x - \frac{1}{x} = 3$

Multiplying both sides by x, we get:

$$x^2 - 1 = 3x$$

$$\Rightarrow \quad x^2 - 3x - 1 = 0$$

This is a quadratic equation.

Here, $a = 1, b = -3$ and $c = -1$

$$\therefore \quad x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$= \frac{3 \pm \sqrt{9 + 4}}{2} = \frac{3 \pm \sqrt{13}}{2}$$

$$\Rightarrow \text{Either } x = \frac{\mathbf{3 + \sqrt{13}}}{\mathbf{2}} \text{ or } x = \frac{\mathbf{3 - \sqrt{13}}}{\mathbf{2}}$$

Refer to the links : https://www.youtube.com/watch?v=BbeRP04pQIM

https://www.youtube.com/watch?v=AAcknrC0QJA

(ii) Forming a Quadratic Equation in a Given Situation:

A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution:

Total distance travelled = 360 km

Let uniform speed be x km/h

Then, increased speed = (x + 5) km/h

According to question,

$$\frac{360}{x} - \frac{360}{x+5} = 1 \qquad \left\{\text{Time} = \frac{\text{Distance}}{\text{Speed}}\right\}$$

$$\Rightarrow \quad \frac{360(x+5) - 360x}{x(x+5)} = 1$$

$$\Rightarrow \quad 360x + 1800 - 360x = x(x+5)$$

$$\Rightarrow \quad 1800 = x^2 + 5x$$

$$\Rightarrow \quad x^2 + 5x - 1800 = 0 \quad \Rightarrow \quad x^2 + 45x - 40x - 1800 = 0$$

$$\Rightarrow \quad x(x+45) - 40(x+45) = 0 \quad \Rightarrow \quad (x-40)(x+45) = 0$$

$$\Rightarrow \quad x - 40 = 0 \text{ or } x + 45 = 0$$

$$\Rightarrow \quad x = 40 \text{ or } x = -45 \text{ (rejected)}$$

$\therefore$ Speed of the train = 40 km/h

Refer to the following links for more word problems :

https://www.youtube.com/watch?v=yHDqZE7XyHA

https://www.youtube.com/watch?v=f2lxRLycnY8

(iii) Nature of Roots

The Discriminant

The discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by $b^2 - 4ac$.
The symbol, Δ is sometimes used for the discriminant.

Note that the discriminant is the part of the quadratic formula that is under the square root sign.

By examining the value of the discriminant we can determine the number and nature of the roots.

If the discriminant is zero	$b^2 - 4ac = 0$	there is one (repeated) rational root
If the discriminant is positive	$b^2 - 4ac > 0$	there are two real roots
If the discriminant is negative	$b^2 - 4ac < 0$	there are no real roots

If the discriminant is a perfect square, such as 49 or 100, then the roots will be rational (fractional) numbers.

Examples:

	example 1	example 2	example 3
Equation	$y = (x + 3)^2$ $= x^2 + 6x + 9$	$y = x^2 - 5x + 6$	$y = -x^2 + x - 2$
a, b and c	a = 1, b = 6, c = 9	a = 1, b = -5, c = 6	a = -1, b = 1, c = -2
Discriminant	$b^2 - 4ac = 6^2 - 4x1x9$ $= 0$ Discriminant = 0 (i.e. Zero)	$b^2 - 4ac = (-5)^2 - 4x1x6$ $= 1$ Discriminant = 1 (i.e. Positive)	$b - 4ac = (1)^2 - 4x-1x-2$ $= -7$ Discriminant = -7 (i.e. Negative)
Number and nature of the roots	There is one repeated real root	There are two real roots	There are no real roots

Refer to the link : https://www.youtube.com/watch?v=yHDqZE7XyHA

Q Find the value of k for which the quadratic equation has two equal roots.

$$kx(x-2) + 6 = 0$$
$$\Rightarrow \quad kx^2 - 2kx + 6 = 0$$

This is of the form $ax^2 + bx + c = 0$,

where $a = k, b = -2k$ and $c = 6$

Discriminant, $D = b^2 - 4ac$

$$= (-2k)^2 - 4 \times k \times 6 = 4k^2 - 24k$$

For equal roots, $D = 0$

$$\Rightarrow \quad 4k^2 - 24k = 0 \quad \Rightarrow \quad k(4k - 24) = 0$$
$$\Rightarrow \quad k = 0 \text{ (not possible)} \quad \text{or} \quad 4k - 24 = 0$$
$$\Rightarrow \quad 4k = 24$$
$$\Rightarrow \quad k = \frac{24}{4} = 6$$

Step 4 : Points to Remember

1) A quadratic equation always has two roots.
2) A given daily life situation will be feasible (Ex 4.4 - Q 3 to 5) if the quadratic equation so formed has real roots.

ASSIGNMENT

Do NCERT Ex 4.1 and 4.2 in the CW/HW register.

NOTE

1. Refer to the following links to practice more questions:

a) https://diksha.gov.in/play/collection/do_31292439596866764812 58?referrer=utm_source%3Ddiksha_mobile%26utm_content%3Ddo_31292439596866764812 58%26utm_campaign%3Dshare_content

b) From Khan Academy Assignments

https://www.khanacademy.org/math/in-in-grade-10-ncert

c) www.examfear.com

d) http://www.ei-india.com/mindspark-math (free trial for 60 days)