SEQUENCES AND SERIES PART-4

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Sum to N terms of a G.P

- Let the first term of a G.P be 'a' and the common ratio be 'r'. We denote the sum to first n terms of G.P by S_n. Then

$$S_n = a + ar + ar^2 + ar^3 + \ldots\ldots + ar^{(n-1)} \ , r \neq 1$$

OR

$$S_n = \frac{a_1(1-r^n)}{1-r}, r \neq 1,$$

Watch videos : https://youtu.be/RYeRq2N9R94

https://youtu.be/4baQj2-BZHM

Sum to N terms of a G.P

Examples :

Q1) Find the sum of the first 8 terms of the geometric series if $a_1 = 1$ and $r = 2$.

$$S_8 = \frac{1(1-2^8)}{1-2} = 255$$

Q2) Find S_{10} of the geometric sequence $24, 12, 6, \cdots$.

First, find r.

$$r = \frac{r_2}{r_1} = \frac{12}{24} = \frac{1}{2}$$

Now, find the sum:

$$S_{10} = \frac{24\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}} = \frac{3069}{64}$$

Sum to N terms of a G.P

Q3)

Evaluate.

$$\sum_{n=1}^{10} 3(-2)^{n-1}$$

(You are finding S_{10} for the series $3 - 6 + 12 - 24 + \cdots$, whose common ratio is -2.)

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_{10} = \frac{3\left[1-(-2)^{10}\right]}{1-(-2)} = \frac{3(1-1024)}{3} = -1023$$

Infinite Geometric Series

An infinite geometric series is the sum of an infinite geometric sequence . This series would have no last term. The general form of the infinite geometric series is $a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \ldots$, where a_1 is the first term and r is the common ratio.

We can find the sum of all finite geometric series. But in the case of an infinite geometric series when the common ratio is greater than one, the terms in the sequence will get larger and larger and if you add the larger numbers, you won't get a final answer. The only possible answer would be infinity. So, we don't deal with the common ratio greater than one for an infinite geometric series.

If the common ratio r lies between -1 to 1 , we can have the sum of an infinite geometric series. That is, the sum exits for $|r| < 1$.

The sum S of an infinite geometric series with $-1 < r < 1$ is given by the formula,

$$S = \frac{a_1}{1-r}$$

Infinite Geometric Series

The sum of infinite terms of a G.P can also be symbolically represented by

$$S_\infty = \sum_{n=1}^{\infty} ar^{n-1} = \frac{a_1}{1-r}, -1 < r < 1$$

Where

S_∞ :sum to infinite terms

a : first term of a G.P

r : constant ratio

n : term number

Watch videos:

https://youtu.be/b-7kCymoUpg

https://youtu.be/q8OhFQIpIIs

Infinite Geometric Series

Examples:

Q1) Find the sum of the infinite geometric sequence

$27, 18, 12, 8, \cdots.$

First find r:

$$r = \frac{a_2}{a_1} = \frac{18}{27} = \frac{2}{3}$$

Then find the sum:

$$S = \frac{a_1}{1-r}$$

$$S = \frac{27}{1-\frac{2}{3}} = 81$$

Infinite Geometric Series

Q2) Find the sum of the infinite geometric sequence

$8, 12, 18, 27, \cdots$ if it exists.

First find r:

$$r = \frac{a_2}{a_1} = \frac{12}{8} = \frac{3}{2}$$

Since $r = \frac{3}{2}$ is not less than one the series has no sum.

TASK

Find the sum to indicated number of terms in each of the geometric progressions in Exercises 7 to 10:

7. 0.15, 0.015, 0.0015, ... 20 terms.

8. $\sqrt{7}$, $\sqrt{21}$, $3\sqrt{7}$, ... n terms.

9. 1, $-a$, a^2, $-a^3$, ... n terms (if $a \neq -1$).

10. x^3, x^5, x^7, ... n terms (if $x \neq \pm 1$).

11. Evaluate $\sum_{k=1}^{11} (2+3^k)$.

12. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

13. How many terms of G.P. 3, 3^2, 3^3, … are needed to give the sum 120?

14. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.

15. Given a G.P. with $a = 729$ and 7^{th} term 64, determine S_7.

16. Find a G.P. for which sum of the first two terms is -4 and the fifth term is 4 times the third term.