BAL BHARATI PUBLIC SCHOOL
PITAMPURA
SESSION 2020-21 CLASS 11 PHYSICS

Chapter–2: Units and Measurements part 3

For this lecture topics are
Dimensional analysis and its remaining applications.

Kindly do the assignment in your physics registers

MAIN POINTS OF THE CHAPTER

Uses of Dimensional Equations
The dimensional equations have got the following uses:

1. **To check the dimensional correctness of a given equation**
2. **To convert a physical quantity from one system of units to another**
3. **To derive the relation between various physical quantities.**
4. **To find the dimension of constants in a given relation**

APPLICATION 2
To convert a physical quantity from one system of units to another
The dimensions of a physical quantity are independent of the system of units used to measure the quantity in.
Like 10m=100 cm
Or n1u1=n2u2
Here n1 and n2 are called numerical values and u1 and u1 are the units in the 2 systems.

Let us suppose that M1 , L1 and T1 and M2 , L2 and T2 are the fundamental quantities in two different systems of units.
We will measure a quantity Q (say) in both these systems of units.

Suppose, a, b, c be the dimensions of the quantity respectively.
In the first system of units, $Q = n_1 u_1 = n_1 [M_1^a L_1^b T_1^c]$
In the second system of units, Q = $n_2 u_2 = n_2 [M_2^a L_2^b T_2^c]$

As $n_1 u_1 = n_2 u_2$

$n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$
Substitution of the respective values will give the value of n1 or n2

$$n_2 = n_1 \frac{[M_1^a L_1^b T_1^c]}{M_2^a L_2^b T_2^c}]$$

SOLVED EXAMPLE

Convert 1 J to erg.

Sol. Joule is the S.I. unit of work. Let this be the first system if units. Also erg is the unit of work in the CHS system of units. This will be the second system of units. Also n1 = 1J and we have to find the value of n2

$n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$

The dimensional formula of work is $[M^1 L^2 T^{-2}]$. As a result
a = 1, b = 2 and c = -2.

Also M1 = 1 kg , L1 = 1 m, T1=1 s and M2 = 1 g ,L2 = 1 cm, T2=1 s
Solving and substituting

$n_2 = n_1 \frac{[M_1^a L_1^b T_1^c]}{M_2^a L_2^b T_2^c}]$
n2 = 10^7
Hence, 1J = 10^7 erg

Solved example 2 from NCERT
A calorie is a unit of heat or energy and it equals about 4.2 J where 1J = 1 kg $m^2 s^{-2}$. Suppose we employ a system of units in

which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γ s. Show that a calorie has a magnitude $4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^{2}$ in terms of the new units.

Answer: Considering the unit conversion formula,

$n_1U_1 = n_1U_2$

$n_1[M_1^aL_1^bT_1^c] = n_2[M_2^aL_2^bT_2^c]$

Given here, 1 Cal = 4.2 J = 4.2 kg m^2 s^{-2}.

$n_1 = 4.2$, $M_1 = 1$kg, $L_1 = 1$m, $T_1 = 1$ sec

and

$n_2 = ?$, $M_2 = \alpha$ kg, $L_2 = \beta$m, $T_2 = \gamma$ sec

The dimensional formula of energy is = $[M^1L^2T^{-2}]$

⇒ a = 1, b =1 and c = -2 Putting these values in above equation,

$n_2= n_1[M_1/M_2]^a[L_1/L_2]^b[T_1/T_2]^c$

$= n_1[M_1/M_2]^1[L_1/L_2]^2[T_1/T_2]^{-2}$

$= 4.2[1Kg/\alpha\ kg]^1[1m/\beta m]^2[1sec/\gamma\ sec]^{-2} = 4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2$

Assignment for application 2

1 Convert a power of 500W into a system of CGS units.

2 Convert 60J/sec to a system which has 500g, 1cm and 1 minute as fundamental units

3 Convert 100 J/sec to a system which has 100g, 1mm and 1 minute as fundamental units.

4 Convert 1 N into the CGS system.

5 Find the value of G in the CGS system.

6 Find the value of 1 dyne in a system of units which has 100m, 1g and 30 sec as fundamental units.

7 Density of mercury is 13.6g/cc. find its value in SI system.

8 Surface tension of water is 7.2 dyne/cm ,find value in SI system.

APPLICATION 3

To derive the relation between various physical quantities.

The idea of dimensional consistency can be sometimes used to deduce relation among the physical quantities. This can be done, if we the dependence of the physical quantity on other two or more physical quantities or linearly dependent variables are known.
Let us try this example.

Consider a simple pendulum, having a bob attached to a string that oscillates under the action of force of gravity as shown in the figure below. Suppose that the period of oscillation (T)of the simple pendulum depends on its length (l),mass of the bob (m) and acceleration due to gravity (g).Derive the expression for its time period using the method of dimensions.

The time period(T) is dependent on the quantities l, g and m and T can be written as the product of l, g ,m :

$$T= k\, l^x g^y m^z \qquad \text{---------------eq(1)}$$

Where

k is a dimensionless constant and x, y and z are the exponents.
Now applying the dimensions on both the sides,

$$[L^0M^0T^1] = [L^1]^x[L^1T^{-2}]^y[M^1]^z$$

Since $L^0=1$; $M^0=1$, we have included on the LHS for convenience. In the RHS we have applied the dimensions of mass, acceleration due to gravity and length.
Now,

$$[L^0M^0T^1] = [L]^x[L^y][T^{-2y}][M^z]$$

$$=[L]^{x+y}[T]^{-2y}[M]^z$$

Equating the exponents of the dimensions on both the sides,
x+y=0 ; -2y=1; and z=0

Therefore, x=-y and x=½y; x= $\frac{1}{2}$,y= $-\frac{1}{2}$,z=0

Substituting the values of x,y,z in equation (1),

$$T=k\, l^{½} g^{½} m^{0}$$

(i.e) $$T=k\sqrt{\frac{l}{g}}$$

Now the expression for the period of oscillation is deduced, but the value of the constant cannot be found.

The value of k= 2π and now the above equation becomes,

$$T=2π\sqrt{\frac{l}{g}}$$

Dimensional analysis is thus useful in obtaining the relationship between those physical quantities which are interdependent.

APPLICATION 4

4 To find the dimension of constants in a given relation

Watch this video

https://www.youtube.com/watch?v=iBRhAZ4sZzI

SOLVED EXAMPLE

Hooke's law states that the force, F, in a spring extended by a length x is given by F = −kx.
According to Newton's second law F = ma, where m is the mass and a is the acceleration. Calculate the dimension of the spring constant k.

Answer: Given, F = -kx

⇒ k = – F/x

F = ma, the dimensions of force is:

$[F] = ma = [M^1L^0T^0].[M^0L^1T^{-2}] = [M^1L^1T^{-2}]$

Therefore, dimension of spring constant (k) is:

$[k] = [F]/[x] = [M^1L^1T^{-2}].[M^0L^{-1}T^0] = [M^1L^0T^{-2}]$ or $[MT^{-2}]$ …

Solved example (NCERT):

A famous relation in physics relates 'moving mass' m to the 'rest mass' mo of a particle in terms of its speed v and the

speed of light, c. (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes $m = \left(\frac{m_0}{1-v^{2)^{1/2}}}\right)$

Answer:

Dimension of m (mass) = $[M^1L^0T^0]$

Dimension of m_0 (mass) = $[M^1L^0T^0]$

Dimension of v (velocity) = $[M^0L^1T^{-1}]$

∴ Dimension of v^2 = $[M^0L^2T^{-2}]$

Dimension of c (velocity) = $[M^0L^1T^{-1}]$

Applying principle of homogeneity of dimensions, [LHS] = [RHS] = $[M^1L^0T^0]$

⇒ The equation $(1- v^2)^{1/2}$ must be dimension less, which is possible if we have the expressions as:

$(1 - v^2/c^2)$ The equation after placing 'c'

$$m = \frac{m_0}{(1 - \frac{v2}{c2})^{1/2}}$$

SOLVED EXAMPLE (NCERT):

A book with many printing errors contains four different formulas for the displacement y of a particle undergoing a certain periodic motion:

(a) y = a sin 2π t/T

(b) y = a sin vt

(c) y = (a/T) sin t/a

(d) y = (a 2) (sin 2πt / T + cos 2πt / T)

(a = maximum displacement of the particle, v = speed of the particle. T = time-period of motion). Rule out the wrong formulas on dimensional grounds.

Answer:

Given,Dimension of a = displacement = $[M^0L^1T^0]$

Dimension of v (speed) = distance/time = $[M^0L^1T^{-1}]$

Dimension of t or T (time period) = $[M^0L^0T^1]$

Trigonometric function **sine** is a ratio, hence it must be dimensionless.

(a) $y = a \sin 2\pi t/T$ (**correct**)
Dimensions of RHS = $[L^1] \sin([T].[T^{-1}])$ = $[M^0L^1T^0]$ = LHS (eqation is correct).
(b) $y = a \sin vt$ (wrong)
RHS = $[L^1] \sin([LT^{-1}] [T^1])$ = $[L^1] \sin([L])$ = wrong, since trigonometric function must be dimension less.
(c) $y = (a/T) \sin t/a$ (wrong ✗)
RHS = $[L^1] \sin([T].[L^{-1}])$ = $[L^1] \sin([TL^{-1}])$ = wrong, sine function must be dimensionless.
(d) $y = (a\ 2) (\sin 2\pi t / T + \cos 2\pi t / T)$ (**correct**)
RHS = $[L^1] (\sin([T].[T^{-1}] + \cos([T].[T^{-1}]) = [L^1] (\sin(M^0L^1T^0) + \cos(M^0L^1T^0))$
= $[L^1]$ = RHS = equation is dimensionally correct.

Solved example
The kinetic energy K of a rotating body depends on its moment of inertia I and its angular speed ω. Considering the relation to be $K = kI^a\omega^b$ where k is dimensionless constant. Find a and b. Moment of Inertia of a sphere about its diameter is $(2/5)Mr^2$
Answer:
Dimensions of Kinetic energy K = $[M^1L^2T^{-2}]$
Dimensions of Moment of Inertia (I) = $[(2/5)Mr^2] = [ML^2T^0]$
Dimensions of angular speed $\omega = [\theta/t] = [M^0L^0T^{-1}]$
Applying principle of homogeneity in dimensions in the equation $K = kI^a\omega^b$
$[M^1L^2T^{-2}] = k ([ML^2T^0])^a([M^0L^0T^{-1}])^b$
$[M^1L^2T^{-2}] = k [M^aL^{2a}T^{-b}]$
⇒ a = 1 and b = 2
⇒ $K = kI\omega^2$

Dimensional constants are constants which possess dimensions are called dimensional constants. E.g. Planck' Constant.

Dimensional variables are those physical quantities which possess dimensions but do not have a fixed value are called dimensional variables. E.g. Displacement, Force, velocity etc.

Limitation of Dimensional analysis

Following are the limitations of the dimensional analysis.

1. It does not give information about the dimensional constant.
2. if a quantity depends on more than three factors having dimension, the formula cannot be derived.
3. We cannot derive the formulae containing trigonometric function, exponential functions, logarithmic function, etc.
4. The exact form of relation cannot be developed when there are more than one part in any relation.
5. It gives no information whether a physical quantity is scalar or vector.

ASSIGNMENT for applications 3,4

1 Assuming that the period of vibration of a tuning fork depends upon the length of the prongs and on the density and Young's modulus of the material, Find by the method of dimensions a formula for the period of vibration.

2 If v,a ,x and t indicate velocity, acceleration, distance and time respectively ,find values for m and n for the following parts.

(a) $x^n = v^m / t^2$ (b) $t = x^m a^n$

3 If Force = (x/density) is dimensionally correct, the dimension of x are

(A) MLT^{-2} (B) MLT^{-3} (C) ML^2T^{-3} (D) $M^2L^{-2}T^{-2}$

4 If velocity v, density p and frequency f are taken as fundamental quantities then express linear momentum and surface tension in terms of these quantities.

5 If v stands for velocity of sound, E is Modulus of elasticity and **d** the density, then find **x** in the equation $\mathbf{v = (d/E)^x}$

A 1 **B** ½ **C** 2 **D** -½

6 Write the dimensions of a and b in the relation P = $a\,x + bt^2$ Where P is pressure, x is distance and t is time.

7 Assuming that the mass of the largest stone that can be moved by a flowing river depends upon v, velocity of flow, p, density of water and g acceleration due to gravity. Derive an expression for mass using dimensions

8 Force of viscosity F acting on a spherical body moving through a fluid depends upon its velocity (v), radius (r) and co-efficient of viscosity 'η' of the fluid. Using method of dimensions obtain an expression for 'F'.

9 Wavelength λ associated with a moving particle depends upon its mass m, its velocity v and Planck's constant. h. Find dimensionally the relation between these quantities.

10 Write the dimensions of a/b in the relation F = a$\sqrt{x}$ + bt^2 Where F is force, x is distance and t is time.

11 Derive by the method of dimensions, an expression for the energy of a body executing S.H.M., assuming that this energy depends upon; the mass (m), the frequency (v) and the amplitude of vibration (r).

12 The magnitude of force experienced by an object moving with speed v is given by
$F=kv^2$. Find dimensions of k.

13 The escape velocity v of a body depends on:
(i) the acceleration due to gravity 'g' of the planet,
(ii) the radius R of the planet. Establish dimensionally the relation for the escape velocity.