BAL BHARATI PUBLIC SCHOOL
PITAMPURA
SESSION 2020-21 CLASS 11 PHYSICS

Chapter–2: Units and Measurements part 2

For this lecture topics are
Dimensions of physical quantities, dimensional analysis and its applications.

Kindly do the assignment in your physics registers

MAIN POINTS OF THE CHAPTER

Dimensions of physical Quantities

The dimensions of a physical quantity are the powers to which the base quantities are raised to represent that quantity.

The dimensions give the nature of the physical quantity.
For example, metres, foot, inches are different units of length but they have the same dimension length and they all measure length. In the same way kilograms and pounds measure the mass and they share the dimension mass. Usually the dimension of a physical is represented in square brackets [ ]

The expression which shows how and which of the base quantities represent the dimensions of a physical quantity is called the *dimensional formula* of the given physical quantity
For eg *dimensional formula for length is* [L]

The equation obtained by equating a physical quantity with its dimensional formula is called the **dimensional equation** of the physical quantity. Eg
Length =[L]

The base or fundamental dimensions for the fundamental physical quantities are

Physical Quantity	***Dimension***
Length	[L]
Mass	[M]
Time	[T]
Electric current	[A]
Temperature	[K]
Luminous intensity	[Cd]
Amount of substance	[mol]

All other physical quantities represented by derived units can be expressed using the combination of the fundamental dimensions.

For example, we will derive the dimension for the area. It is expressed as the product of lengths (Area = Length × Breadth)

Applying the dimensions, we get

$[L] \times [L] = [L]^2 = [L^2]$ which means that the area has two dimensions of length. The exponent or the power to which the base quantity is raised represents the number of dimensions of that quantity present.

Example : Find the dimensional formula for force

$$\begin{aligned} \text{Force} &= \text{mass} \times \text{acceleration} \\ &= \text{mass} \times (\text{length})/(\text{time})^2 \\ &= [M][L]/[T^2] \\ &= [M][L][T^{-2}] \end{aligned}$$

Thus force has one dimension in mass, one dimension in length and -2 dimensions in time.

Try it yourself

Derive the dimensional formula for

a) Volume

b) Density
c) Velocity
d) Acceleration
e) Force
f) Momentum
g) Work

- Energy
- K.E
- P.E
- Power
- Frequency
- Gravitational constant
- Pressure
- Wavelength
- Efficiency
- Angle
- Trigonometric ratios

Look at appendix A 9 at the back of NCERT book 1.
http://ncert.nic.in/textbook/textbook.htm?keph1=a1-8

S.No	Physical quantity	Relationship with other physical quantities	Dimensions	Dimensional formula
1.	Area	Length × breadth	$[L^2]$	$[M^0 L^2 T^0]$
2.	Volume	Length × breadth × height	$[L^3]$	$[M^0 L^3 T^0]$
3.	Mass density	Mass/volume	$[M]/[L^3]$ or $[M L^{-3}]$	$[M L^{-3} T^0]$
4.	Frequency	1/time period	$1/[T]$	$[M^0 L^0 T^{-1}]$
5.	Velocity, speed	Displacement/time	$[L]/[T]$	$[M^0 LT^{-1}]$
6.	Acceleration	Velocity /time	$[LT^{-1}]/[T]$	$[M^0 LT^{-2}]$
7.	Force	Mass × acceleration	$[M][LT^{-2}]$	$[M LT^{-2}]$
8.	Impulse	Force × time	$[M LT^{-2}][T]$	$[M LT^{-1}]$
9.	Work, Energy	Force × distance	$[MLT^{-2}][L]$	$[M L^2 T^{-2}]$
10.	Power	Work/time	$[ML^2 T^{-2}]/[T]$	$[M L^2 T^{-3}]$
11.	Momentum	Mass × velocity	$[M][LT^{-1}]$	$[M LT^{-1}]$
12.	Pressure, stress	Force/area	$[M LT^{-2}]/[L^2]$	$[ML^{-1}T^{-2}]$
13.	Strain	$\frac{\text{Change in dimension}}{\text{Oringinal dimension}}$	$[L]/[L]$ or $[L^3]/[L^3]$	$[M^0 L^0 T^0]$
14.	Modulus of elasticity	Stress/strain	$\frac{[M L^{-1} T^{-2}]}{[M^0 L^0 T^0]}$	$[M L^{-1} T^{-2}]$
15	Surface tension	Force/length	$[MLT^{-2}]/[L]$	$[ML^0 T^{-2}]$
16.	Surface energy	Energy/area	$[ML^2 T^{-2}]/[L^2]$	$[ML^0 T^{-2}]$
17.	Velocity gradient	Velocity/distance	$[LT^{-1}]/[L]$	$[M^0 L^0 T^{-1}]$
18.	Pressure gradient	Pressure/distance	$[ML^{-1}T^{-2}]/[L]$	$[ML^{-2} T^{-2}]$
20.	Coefficient of viscosity	Force/area × velocity gradient	$\frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]}$	$[ML^{-1} T^{-1}]$
21.	Angle, Angular displacement	Arc/radius	$[L]/[L]$	$[M^0 L^0 T^0]$
22.	Trigonometric ratio (sinθ, cosθ, tanθ, etc.)	Length/length	$[L]/[L]$	$[M^0 L^0 T^0]$
23.	Angular velocity	Angle/time	$[L^0]/[T]$	$[M^0 L^0 T^{-1}]$

24.	Angular acceleration	Angular velocity/time	$[T^{-1}]/[T]$	$[M^0L^0T^{-2}]$
25.	Radius of gyration	Distance	$[L]$	$[M^0LT^0]$
26.	Moment of inertia	Mass × (radius of gyration)^2	$[M]\,[L^2]$	$[ML^2\,T^0]$
27.	Angular momentum	Moment of inertia × angular velocity	$[ML^2]\,[T^{-1}]$	$[ML^2\,T^{-1}]$
28.	Moment of force, moment of couple	Force x distance	$[MLT^{-2}]\,[L]$	$[ML^2\,T^{-2}]$
29.	Torque	Angular momentum/time, Or Force × distance	$[ML^2\,T^{-1}]\,/\,[T]$ or $[MLT^{-2}]\,[L]$	$[ML^2\,T^{-2}]$
30.	Angular frequency	2π × Frequency	$[T^{-1}]$	$[M^0L^0T^{-1}]$
31.	Wavelength	Distance	$[L]$	$[M^0LT^0]$
32.	Hubble constant	Recession speed/distance	$[LT^{-1}]/[L]$	$[M^0L^0T^{-1}]$
36.	Critical velocity	$\frac{\text{Reynold's number} \times \text{coefficient of viscocity}}{\text{Mass density} \times \text{radius}}$	$\frac{[M^0L^0T^0][ML^{-1}\,T^{-1}]}{[ML^{-3}][L]}$	$[M^0LT^{-1}]$
37.	Escape velocity	(2 × acceleration due to gravity × earth's radius)$^{1/2}$	$[LT^{-2}]^{1/2} \times [L]^{1/2}$	$[M^0LT^{-1}]$
38.	Heat energy, internal energy	Work (= Force × distance)	$[MLT^{-2}]\,[L]$	$[ML^2\,T^{-2}]$
39.	Kinetic energy	(1/2) mass × (velocity)^2	$[M]\,[LT^{-1}]^2$	$[ML^2T^{-2}]$
40	Potential energy	Mass × acceleration due to gravity × height	$[M]\,[LT^{-2}]\,[L]$	$[ML^2\,T^{-2}]$
44.	Gravitational constant	$\frac{\text{Force} \times (\text{distance})^2}{\text{mass} \times \text{mass}}$	$\frac{[MLT^{-2}][L^2]}{[M]\,[M]}$	$[M^{-1}L^3T^{-2}]$
45.	Planck constant	Energy/frequency	$[ML^2\,T^{-2}]\,/[T^{-1}]$	$[ML^2\,T^{-1}]$

ASSIGNMENT

You are required to write the dimensional formulae of the above physical quantities in your register.

The ones left out are not required to be done at this point in time.

Dimensional analysis and its applications

Dimensional analysis is a method to find or check the relations between the physical quantities.
The basis dimensional analysis revolves around the concept of *homogeneity.*
When we write an equation involving the relationship between physical quantities in the form of mathematical expression we must make sure that the units on both sides are same.

If the units do not match, it means that the equation is not correct.
The identical units in the numerator and the denominator can be cancelled. Similarly the same can be done for the dimensions of a physical quantity.

Dimensional consistency
We can only combine (add or subtract) the similar physical quantities. For instance, velocity cannot be added to force, or the Temperature added to speed.

The simple principle of dimensional consistency is that the quantities on the right hand and the left hand sides of the equality sign in any given physical law must have the same dimensions or the same combination of the dimensions. This principle is useful in checking the correctness of an equation

Uses of Dimensional Equations
The dimensional equations have got the following uses:

1. **To check the dimensional correctness of a given equation**
2. **To convert a physical quantity from one system of units to another**
3. To derive the relation between various physical quantities.
4. To find the dimension of constants in a given relation

APPLICATION 1
To check the dimensional correctness of a given equation

Now let us consider the famous equation by Einstein

$$E=mc^2,$$

where E is the energy of a body and m is its mass and c is the velocity of light in vacuum.

The dimensions of energy are $[M]\ [L^2]\ /\ [T^2]$ and velocity $[L]\ /[T]$

Equating the LHS and RHS,

$$[M]\ [L^2]\ /\ [T^2] = [M]\ ([L]\ /[T])^2$$

$$[M]\ [L^2]\ /\ [T^2] \quad = [M]\ [L^2]\ /[T^2]$$

Notice that the dimensions on both sides are same.

Therefore this equation is a simple, dimensionally consistent equation combining mass, energy and velocity.

TRY THIS

Using the principle of dimensional consistency, find out the correct expression for kinetic energy of a body of mass m and velocity v from the choices given below.

a) $K= m^2v^3$
b) $K= mv$
c) $K=(½)mv^2$
d) $K= (½)mv^2+mv$

Assignment

1 Which of the following quantities has the dimensions $[M^0\ L^0\ T^0]$

A. Density B. Stress C. Strain **D.** Strain Rate

2 [$M^1L^0T^{-2}$] is the dimensional formula of

A Force **B.** Pressure **C** Surface Tension **D** Energy

3 What is the dimensional formula of (a) gravitational constant (b) Surface tension

4 Name a pair of physical quantities which have different natures but same dimensional formula?

5 What is the dimensional formula of (a) Pressure gradient (b) Co-efficient of viscosity.

6 Give two examples of dimension less variables.

7 Name the physical quantities that have dimensional formula [$ML^{-1} T^{-2}$]

8 Check the correctness of the given equations

- **(1)** **P=hdg** where P is pressure,d is density and g is acceleration due to gravity.
- **(2)** $F=mv^2/r$ where F is force, m is mass, v is velocity and r is radius
- **(3)** F=6πηrv where where F is force,η is coefficient of viscosity, v is velocity and r is radius
- **(4)** $R=u^2\sin 2\Theta/g$ where R is distance, u is initial speed , gis acceleration due to gravity
- **(5)** V=u+at
- **(6)** $S=ut+1/2at^2$
- **(7)** $V^2-U^2=2as$ where symbols have usual meanings