AL BHARATI PUBLIC SCHOOL, PITAMPURA, DELHI – 110034

SUBJECT:- MATHEMATICS

Class- IX

CHAPTER:- 2 (PART-7)

TOPIC:-POLYNOMIALS

GUIDELINES

Dear Students
Kindly read the content given below and view the links shared for better understanding.

- **Solve the given NCERT questions in the yellow register provided in the notebook set.**

Link for the chapter : http://ncert.nic.in/textbook/textbook.htm?jemh1=3-15

http://ncert.nic.in/textbook/textbook.htm

Introduction :

Let us recall the following concepts:

The Factor Theorem

If p(x) is a polynomial of degree $n \geq 1$ and a is any real number, then

(i) $(x - a)$ is a factor of p(x) if $p(a) = 0$, and

(ii) $p(a) = 0$, if $(x - a)$ is a factor of p(x).

Factorisation of Polynomials:

A) **BY SPLITTING THE MIDDLE TERM**

You can factorise quadratic polynomials by splitting the middle term as follows:

To begin with, consider a quadratic polynomial $ax^2 + bx + c$, we will write **b as the sum of two numbers** whose **product is 'ac'.**

Case (i) : When **ac** is **positive** :- In this case we have to split ac into two factors either both positive or both negative depending upon the sign of b. If **b is positive both the factors will be positive and if b is negative then both the factors will be negative**. In both these cases sum of both the factors has to be equal to b.

Let's look at these examples to understand this better.

Example1: Factorise $6x^2 + 17x + 5$ by splitting the middle term.

Solution 1: (By Splitting Method): As explained above, if we can find two numbers, 'p' and 'q' such that, p + q = 17 and pq = 6 x 5 = 30, then we can get the factors.

After looking at the factors of 30, we find that numbers '2' and '15' satisfy both the conditions, i.e. p + q = 2 + 15 = 17 and pq = 2 x 15 = 30. So,

$$6x^2 + 17x + 5 = 6x^2 + (2 + 15)x + 5$$
$$= 6x^2 + 2x + 15x + 5$$
$$= 2x(3x + 1) + 5(3x + 1)$$
$$= (3x + 1)(2x + 5).$$

Example 2: Factorise: $y^2 – 5y + 6$.

Solution: Let $p(y) = y^2 – 5y + 6$.

(By Splitting Method):As explained above, if we can find two numbers, 'p' and 'q' such that, p + q = -5 and pq = 1 × 6 = 6, then we can get the factors.

After looking at the factors of 6, we find that numbers '-2' and '-3' satisfy both the conditions, i.e. p + q = -2+ -3 = -5and pq = -2 x -3 = 6. So,

$$y^2 – 5y + 6 = y^2 + (-2 + -3)y + 6$$
$$= y^2 – 2y - 3y + 6$$
$$= y(y - 2) - 3(y - 2)$$
$$= (y – 2)(y – 3)$$

Case(ii) :- When **ac is negative**, then we split ac into two factors in which one will be positive and the other will be negative. The sum of those two factors will be equal to b.

Let's look at these examples to understand this better.

Example1: Factorise: $6x^2 + 13x - 5$ by splitting the middle term.

Solution:(By Splitting Method):As explained above, if we can find two numbers, 'p' and 'q' such that, p + q = 13 and pq = 6 x -5 = -30, then we can get the factors.

After looking at the factors of 30, we find that numbers '-2' and '15' satisfy both the conditions, i.e. p + q = -2 + 15 = 13 and pq = - 2 x 15 =- 30. So,

$6x^2 + 13x - 5 = 6x^2 + (-2 + 15)x - 5$

$= 6x^2 - 2x + 15x - 5$
$= 2x(3x - 1) + 5(3x - 1)$
$= (3x - 1)(2x + 5)$

Example2: Factorise : $8y^2 - 10y - 7$.

Solution: Let $p(y) = 8y^2 - 10y - 7$.

(By Splitting Method): As explained above, if we can find two numbers, 'p' and 'q' such that, $p + q = -10$ and $pq = 8 \times (-7) = -56$, then we can get the factors.

After looking at the factors of -56, we find that numbers '-14' and '4' satisfy both the conditions, i.e. $p + q = -14 + 4 = -10$ and $pq = -14 \times 4 = -56$. So,

$8y^2 - 10y - 7 = 8y^2 + (-14 + 4)y - 7$

$= 8y^2 - 14y + 4y - 7$
$= 2y(4y - 7) + 1(4y - 7)$

$= (2y + 1)(4y - 7)$

B)With the help of Factor Theorem :-

In this case, we find one zero of the polynomial p(x) by trial and error method. As soon as we find one zero say 'a' of the polynomial then $(x - a)$ becomes the factor of p(x) by factor theorem.

Let's understand this with the help of the following examples.

Example 1:Factorise: $y^2 - 5y + 6$

Solution: Let $p(y) = y^2 - 5y + 6$. Now, if $p(y) = (y - a)(y - b)$, the constant term will be ab as seen below,

$p(y) = (y - a)(y - b)$
$= y^2 - by - ay + ab$

On comparing the constants, we get $ab = 6$.

Next, the factors of 6 are 1, 2 and 3.

Now, $p(2) = 2^2 - (5 \times 2) + 6 = 4 - 10 + 6 = 0$. So, $(y - 2)$ is a factor of p(y).

Also, $p(3) = 3^2 - (5 \times 3) + 6 = 9 - 15 + 6 = 0$. So, $(y - 3)$ is also a factor of $y^2 - 5y + 6$.

Therefore, $y^2 - 5y + 6 = (y - 2)(y - 3)$

Reference link : https://youtu.be/0K0KO0t27eY

Example 2: Factorise $x^3 - 2x^2 - x + 2$

Answer : Let $p(x) = x^3 - 2x^2 - x + 2$. To begin with, we will find the factors of the constant '2', which are: 1, 2

By trial, we find that $p(2) = 0$. Hence, we conclude that $(x - 2)$ is a factor of $p(x)$.
So, by removing the common factors, we get $x^3 - 2x^2 - x + 2$

$= x^2(x - 2) - (x - 2) = (x^2 - 1)(x - 2)$
$= (x + 1)(x - 1)(x - 2)$ … [Using the identity $(x^2 - 1) = (x + 1)(x - 1)$]

Therefore, the factors of $x^3 - 2x^2 - x + 2$ are $(x + 1)$, $(x - 1)$ and $(x - 2)$

Example 3: Factorise $x^3 - 23x^2 + 142x - 120$

Answer : Let $p(x) = x^3 - 23x^2 + 142x - 120$. To begin with, we will find the factors of the constant '– 120', which are: ±1, ±2, ±3, ±4, ±5, ±6, ±8, ±10, ±12, ±15, ±20, ±24, ±30, ±40, ±60 and ±120 Further, by trial, we find that $p(1) = 0$. Hence, we conclude that $(x - 1)$ is a factor of $p(x)$. Also, we see that

$[x^3 - 23x^2 + 142x - 120] = x^3 - x^2 - 22x^2 + 22x + 120x - 120$

So, by removing the common factor, we get

$x^3 - 23x^2 + 142x - 120 = x^2(x - 1) - 22x(x - 1) + 120(x - 1)$

Further, taking '$x - 1$' common, we get $x^3 - 23x^2 + 142x - 120 = (x - 1)(x^2 - 22x + 120)$

Therefore, $x^3 - 23x^2 + 142x - 120 = (x - 1)(x^2 - 22x + 120)$
Also, note that if we divide $p(x)$ by '$x - 1$', then the result will be $(x^2 - 22x + 120)$
Going on, $x^2 - 22x + 120$ can be factorised further. So, by splitting the middle term, we get:
$x^2 - 22x + 120 = x^2 - 12x - 10x + 120$

as [(– 12 – 10 = – 22) and {(–12)(–10) = 120}]

$= x(x – 12) – 10(x – 12)$

$= (x – 12) (x – 10)$

Therefore, we have $x^3 – 23x^2 – 142x – 120 = (x – 1)(x – 10)(x – 12)$

Reference link : https://youtu.be/MRznpstG07s

Key points and important links for reference :

Refer to this link to enhance your knowledge of factorisation of polynomials:
https://youtu.be/lnckGvtqvoo

ASSIGNMENT :-

(To be done in the yellow register.)

1. **Ex 2.4**
   Q 4. ii and iv part
   Q 5. ii and iii part

QUESTIONS FOR PRACTICE

Online practice:

https://in.ixl.com/math/class-ix/factorise-polynomials