

# BAL BHARATI PUBLIC SCHOOL PITAMPURA

SESSION 2020-21

CLASS 12 PHYSICS

## Chapter-1 NCERT Electric Charges and Fields PART 4

### INSTRUCTIONS

1 This is part 4 of the first chapters of class 12<sup>th</sup> physics which consists of the following topics as per CBSE syllabus

Electric Charges; Conservation of charge, Coulomb's law-force between two point charges, forces between multiple charges; superposition principle and continuous charge distribution. **Electric field, electric field due to a point charge, electric field lines, electric dipole, electric field due to a dipole, torque on a dipole in uniform electric field.** Electric flux, statement of Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside).

**In this lecture we are going to study the following topics**

**Electric dipole, electric field due to a dipole, torque on a dipole in uniform electric field, WD to rotate a dipole (from chapter 2)**

2 First 3 chapters of NCERT part 1 have a combined weightage of 16 marks in the CBSE examination.

3 To fully prepare this chapter, you can practice questions from previous year CBSE papers given along here.

4 Kindly do the assignment in your physics registers

### Main points of the chapter

#### Electric Dipole

It is a system of two equal and opposite charges separated by a small distance.

$-q \leftarrow a \text{---} O \text{---} a \rightarrow +q$

Here O is the centre of dipole and a is distance of each charge from O

## Electric Dipole Moment

$$\vec{p} = q \times 2\vec{a}$$

**Electric Dipole Moment** is a vector quantity. Direction of dipole moment vector is from **negative to positive charge** by convention.

Its SI unit is C-m

### Ideal dipole

It is one in which the distance between the charges is infinitely small. Eg water molecule and other such microscopic molecules.

### Polar and non polar molecules

Non-polar molecules are symmetrical in shape and centers of -ve & +ve charges co-incide e.g CO<sub>2</sub> & CH<sub>4</sub>

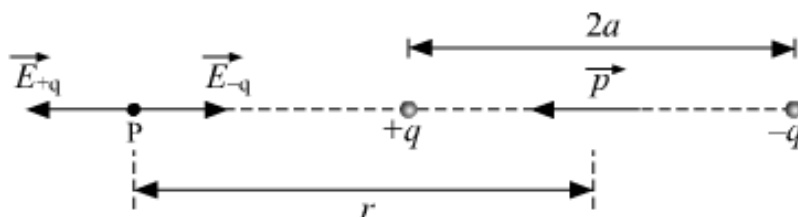
Polar molecules are asymmetrical in shape and centers of -ve & +ve charges do not co-incide eg H<sub>2</sub>O.

## ELECTRIC FIELD DUE TO DIPOLE

### 1 At a point lying on the axial line of the dipole

The axial line of a dipole is the line passing through the positive and negative charges of the electric dipole.

Consider a system of charges (-q and +q) separated by a distance 2a.



Let 'P' be any point on an axis where the EFI is to be determined.

Magnitude of Electric field at P (E<sub>q</sub>) due to +q

$$E_q = \frac{kq}{(r - a)^2}$$

Magnitude of **Electric field at P due to -q (E-q)**

$$E_{-q} = \frac{kq}{(r+a)^2}$$

**Net field at P is given by vector sum of  $E_{-q}$  and  $E_q$**

$$E_p = \frac{kq}{(r-a)^2} - \frac{kq}{(r+a)^2}$$

Simplifying, we get

$$E_p = \frac{4kqra}{(r^2 - a^2)^2}$$

Vectorially , also since  $p = q2a$

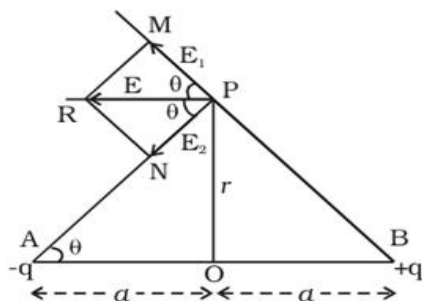
$$\vec{E}_p = \frac{2k\vec{p}r}{(r^2 - a^2)^2}$$

For short dipole ,  $r \gg a$

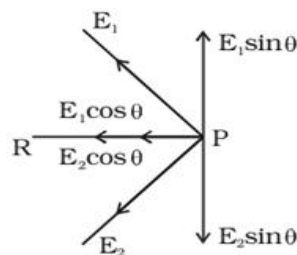
$$\vec{E}_p = \frac{2k\vec{p}}{r^3}$$

## 2 At a point lying on the equatorial line of the dipole

An equatorial line of a dipole is the line perpendicular to the axial line and passing through a point mid way between the charges. Consider a dipole consisting of -q and +q separated by a distance 2a. Consider a point P on the equatorial line.



(a) Electric field at a point on equatorial line



(b) The components of the electric field

Image courtesy physics stack exchange

The resultant intensity is the vector sum of the intensities along PM and PN. E1 and E2 can be resolved into vertical and horizontal components. The vertical components of E1 and E2 cancel each other as they are equal and oppositely directed. It is the horizontal components which add up to give the resultant field.

$$E_1 = E_2 \text{ (in magnitude)} = \frac{kq}{r^2 + a^2}$$

$$E_p = 2E_1 \cos\theta$$

$$E_p = 2 \frac{kq}{r^2 + a^2} \cos\theta$$

$$E_p = \frac{2kq}{r^2 + a^2} \times \frac{a}{(r^2 + a^2)^{1/2}}$$

$$E_p = \frac{2kqa}{(r^2 + a^2)^{3/2}}$$

**Vectorially**

$$\vec{E}_p = \frac{-k\vec{p}}{(r^2 + a^2)^{3/2}}$$

Minus sign indicates that  $\vec{E}_p$  and  $\vec{p}$  are in opposite directions.

**For a short dipole**

$$\vec{E}_p = \frac{-k\vec{p}}{r^3}$$

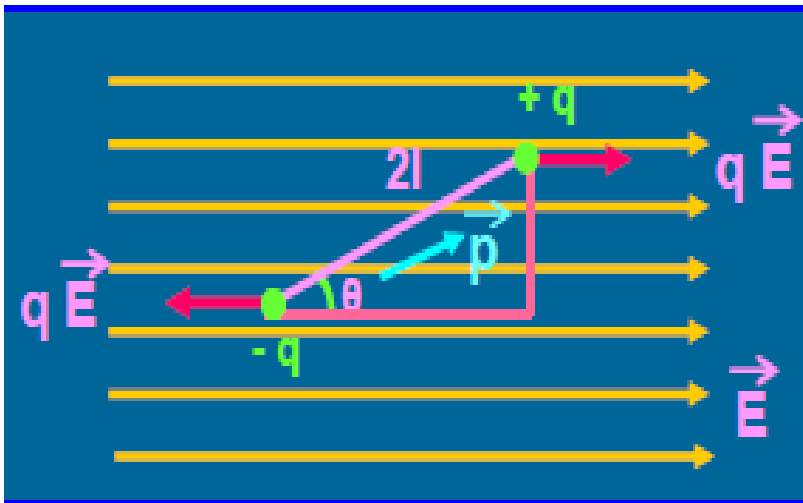
We find that at very far off points i.e.,  $2a < r < \infty$  (short dipole) Electricity intensity at an axial point is twice the electric intensity on the equatorial line.

**Electric dipole in uniform electric field**

When a dipole is placed in an uniform electric field, each charge experience a force ( $F=qE$ ) but in opposite directions.

**There is a force  $qE$  on  $q$  and a force  $-qE$  on  $-q$ . The net force on the dipole is zero, since  $E$  is uniform.**

Hence dipole will not have linear or translatory motion.



*Image courtesy Prayash Mohapatra*

### Torque on dipole

However, the charges are separated, so the forces act at different points, resulting in a torque on the dipole. When the net force is zero, the torque (couple) is independent of the origin.

Its magnitude equals the magnitude of each force multiplied by the arm of the couple (perpendicular distance between the two anti-parallel forces).

$$\begin{aligned}\text{Magnitude of torque} &= q E \times 2 a \sin\theta \\ &= 2 q a E \sin\theta\end{aligned}$$

Its direction is normal to the plane of the paper, coming out of it.

The magnitude of  $\vec{p} \times \vec{E}$  is  $p E \sin\theta$  and its direction is normal to the paper, coming out of it. Thus,

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} \text{ (in vector form)}$$

This torque will tend to align the dipole with the field  $E$ . When  $\vec{p}$  is aligned with  $\vec{E}$ , the torque is zero.

NOTE :

1 If the dipole is parallel to  $E$  the torque is Zero( As angle is zero)

2. Torque is maximum when Dipole is perpendicular to  $E$  and the value of torque is  $pE$ .

3. If a dipole placed in an electric field is allowed to rotate, then it will rotate to align itself to the Electric field.

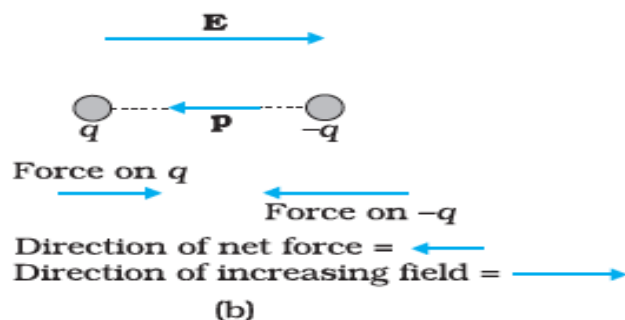
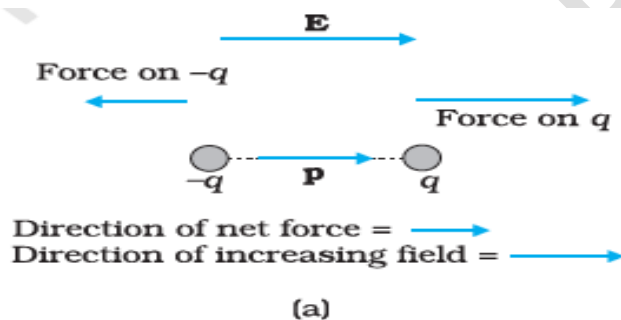
*But when it will reach along the direction of  $E$ , the torque becomes zero. But due to inertia it may overshoot this equilibrium condition and will then starts oscillating about this mean position.*

### ***Dipole in Non-Uniform Electric field***

In case Electric field is non-uniform, magnitude of force on  $+q$  and  $-q$  will be different, hence a net force will be acting on centre of mass of dipole and it will translate in the direction of the net force.

At the same time due to couple of forces acting, a torque will also be acting on it. So it may experience both net force and net torque.

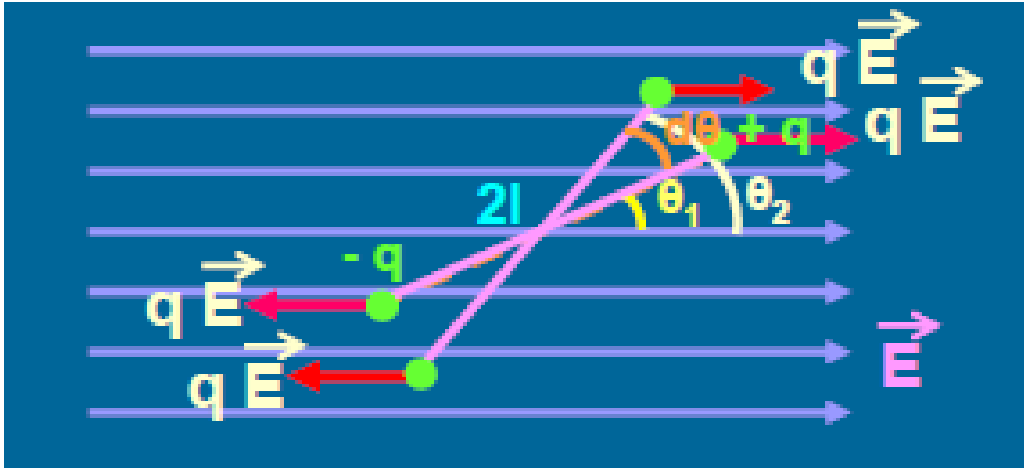
A special situation of the same is given below



### ***Dipole kept parallel to an increasing field NCERT***

### **Work done on an Electric Dipole in Uniform Electric Field:**

Let a dipole be placed in an uniform electric field  $E$ .



(Image courtesy Prayash Mohapatra)

Then it experiences a torque

$$\tau = pE \sin \theta$$

If we rotate the dipole in opposite sense away from the electric field direction, work done

$$dW = \tau d\theta = p E \sin \theta d\theta$$

$$W = \int_{\theta_1}^{\theta_2} p E \sin \theta d\theta$$

$$W = - p E (\cos \theta_2 - \cos \theta_1)$$

If  $\theta_1 = 90^\circ$  and  $\theta_2 = \theta$

P.E in rotating the dipole and inclining it at an angle  $\theta$  is

$$\text{Potential Energy } U = - p E \cos \theta$$

When an electric dipole is placed in a uniform electric field, it experiences torque and tends to align in such a way to attain stable equilibrium.

Case i: If  $\theta = 0^\circ$ , then  $U = - pE$  (Stable Equilibrium)

Case ii: If  $\theta = 90^\circ$ , then  $U = 0$

Case iii: If  $\theta = 180^\circ$ , then  $U = pE$  (Unstable Equilibrium)

## Video links

Prof Walter Lewin's lecture (*same as last time , covers both topics*)

<https://www.youtube.com/watch?v=Pd9HY8iLiCA&t=1163s>

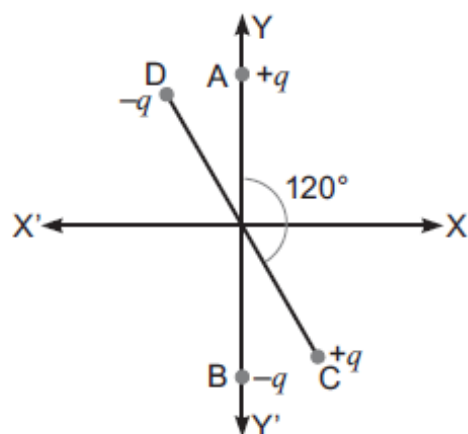
## ASSIGNMENT

1. Why an electric dipole placed in a uniform electric field does not undergo acceleration?
2. What will happen if a dipole is placed in a non – uniform electric field?
3. An electric dipole is placed in an electric field due to a point charge. Will there be a force and torque on the dipole?
4. Name the physical quantities whose SI units are Vm, V/m  
Which of these are vectors?
5. An electric dipole consists of charges  $\pm 16 \times 10^{-19}\text{C}$  separated by a distance of  $3.9 \times 10^{-12}\text{ m}$ . The dipole is placed in a uniform electric field of  $10^5\text{N/C}$ . Find the (i) electric dipole moment (ii) Potential energy of the dipole in the stable equilibrium position.
6. Two point charges  $+3.2 \times 10^{-19}\text{C}$  and  $- 3.2 \times 10^{-19}\text{ C}$  situated at a distance  $2.4 \times 10^{-10}\text{ m}$  apart constitute an electric dipole. What is the work done in rotating the dipole through  $180^\circ$  from the stable equilibrium position in a uniform electric field of  $4 \times 10^5\text{ V/m}$ .
7. Show that no translatory force acts on an electric dipole placed in uniform electric field, Also derive an expression for torque acting experienced by it.
8. An electric dipole consists of two opposite charges each of magnitude  $6 \times 10^{-8}\text{ coulomb}$  separated by  $6.0\text{cm}$ . The dipole is placed in an external electric field of  $5 \times 10^5\text{ N/C}$  .
  - a. What maximum torque will the field exert on the dipole?



b. How much work will an external agent have to do in turning the dipole through 180 degree starting from the position  $\theta = 0$  degree ?

9 Two small identical electrical dipoles  $AB$  and  $CD$ , each of dipole moment ' $p$ ' are kept at an angle of  $120^\circ$  as shown in the figure. What is the resultant dipole moment of this combination? If this system is subjected to electric field  $E$  directed along + X direction, what will be the magnitude and direction of the torque acting on this?



10 An electric dipole  $p$  is placed in a uniform electric field  $E$ . for what angle between  $p$  and  $E$  is the PE of dipole is half its maximum value?

11 Calculate the amount of work done in rotating a dipole, of dipole moment  $5 \times 10^{-8} \text{ cm}$ , from its position of stable equilibrium to the position of unstable equilibrium, in electric field of intensity  $10^4 \text{ N/C}$ .

12 An electric dipole of length 4 cm, when placed with its axis making an angle of  $60^\circ$  with a uniform electric field, experiences a torque of  $4\sqrt{3} \text{ Nm}$ . Calculate the potential energy of the dipole, if it has charge  $\pm 8 \text{ nC}$ .