

# BAL BHARATI PUBLIC SCHOOL PITAMPURA

SESSION 2020-21

CLASS 12 PHYSICS

## Chapter-1 NCERT Electric Charges and Fields PART 2

### INSTRUCTIONS

1 This is one of the most vast and important chapters of class 12<sup>th</sup> physics which consists of the following topics as per CBSE syllabus

*Electric Charges; Conservation of charge, Coulomb's law-force between two point charges, forces between multiple charges; superposition principle and continuous charge distribution. Electric field, electric field due to a point charge, electric field lines, electric dipole, electric field due to a dipole, torque on a dipole in uniform electric field. Electric flux, statement of Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside)*

**In this lecture we are going to do the following topics**

**Forces between multiple charges; superposition principle and continuous charge distribution**

2 First 3 chapters of NCERT part 1 have a combined weightage of 16 marks in the CBSE examination.

3 To fully prepare this chapter , you can practice previous year question papers of CBSE given along here.

4 Kindly do the assignment in your physics registers.

5 This part has mostly numericals which need to be done carefully. A number of links to more problems are also given

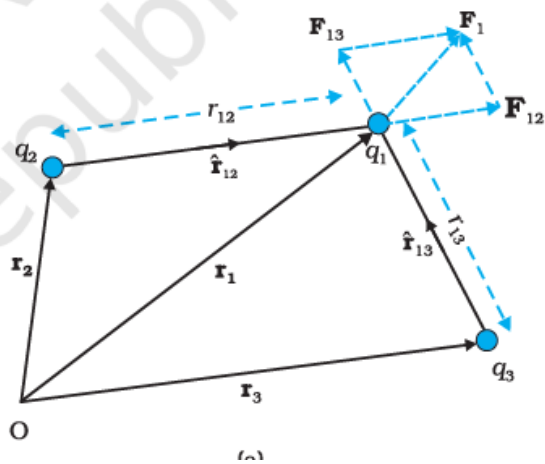
### Main points

#### Principle of Superposition of Charges :

In a system of  $n$  charges, the net electric force on any one individual charge is the vector sum of all individual forces exerted on the charge by all the other charges.

$$\mathbf{F}_1 = \mathbf{F}_{11} + \mathbf{F}_{12} + \mathbf{F}_{13} + \dots + \mathbf{F}_{1n}$$

The vector sum is obtained as usual by parallelogram law of vectors.



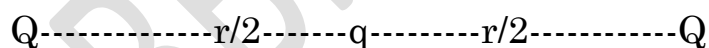
(Diagram courtesy NCERT book)

**Basically since Force is a vector, it can not be added algebraically and vector addition is required to add the vectors.**

**Also, the principle of superposition says that in a system of charges  $q_1, q_2, \dots, q_n$ , the force on  $q_1$  due to  $q_2$  is the same as given by Coulomb's law, i.e., it is unaffected by the presence of the other charges  $q_3, q_4, \dots, q_n$ .**

### SOLVED PROBLEM 1

A charge  $q$  is placed in between 2 equal charges  $Q$  kept at a distance  $r$  on a straight line. Find the value of  $q$  in terms of  $Q$  and  $r$  so that all the three charges are in equilibrium.



As the middle charge is in the centre, it experiences two equal and opposite forces and hence is in equilibrium (this is irrespective of nature of  $q$ , it may be positive or negative).

As per the situation, let's consider the equilibrium of one corner charge. On the left  $Q$ , there are 2 forces acting  $F_{QQ}$  towards left (repulsion between  $Q$  and  $Q$ ) and  $F_{Qq}$  whose direction depends on the sign of  $q$ . At  $Q$ , the  $F_{Qq}$  must be towards right so that net force on  $Q$  is zero.

In magnitude  $F_{Qq} = F_{Qq}$

In terms of direction  $F_{Qq} = -F_{Qq}$

Substituting and solving  $q = -Q/4$

### Solved problem 2 ncert

**Example 1.6** Consider three charges  $q_1, q_2, q_3$  each equal to  $q$  at the vertices of an equilateral triangle of side  $l$ . What is the force on a charge  $Q$  (with the same sign as  $q$ ) placed at the centroid of the triangle, as shown in Fig. 1.9?

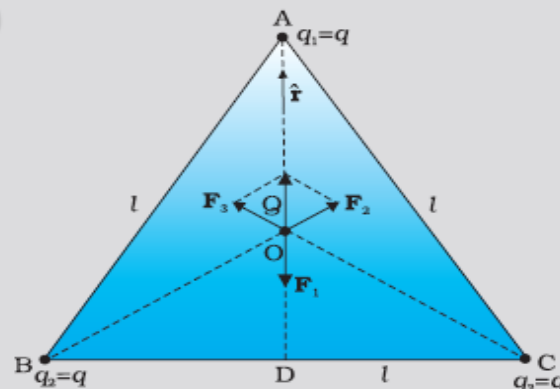


FIGURE 1.9

**Solution** In the given equilateral triangle ABC of sides of length  $l$ , if we draw a perpendicular AD to the side BC,

$AD = AC \cos 30^\circ = (\sqrt{3}/2) l$  and the distance AO of the centroid O from A is  $(2/3) AD = (1/\sqrt{3}) l$ . By symmetry  $AO = BO = CO$ .

Thus,

$$\text{Force } \mathbf{F}_1 \text{ on } Q \text{ due to charge } q \text{ at A} = \frac{3}{4\pi\epsilon_0} \frac{Qq}{l^2} \text{ along AO}$$

$$\text{Force } \mathbf{F}_2 \text{ on } Q \text{ due to charge } q \text{ at B} = \frac{3}{4\pi\epsilon_0} \frac{Qq}{l^2} \text{ along BO}$$

$$\text{Force } \mathbf{F}_3 \text{ on } Q \text{ due to charge } q \text{ at C} = \frac{3}{4\pi\epsilon_0} \frac{Qq}{l^2} \text{ along CO}$$

The resultant of forces  $\mathbf{F}_2$  and  $\mathbf{F}_3$  is  $\frac{3}{4\pi\epsilon_0} \frac{Qq}{l^2}$  along OA, by the parallelogram law. Therefore, the total force on  $Q = \frac{3}{4\pi\epsilon_0} \frac{Qq}{l^2} (\hat{\mathbf{r}} - \hat{\mathbf{r}})$

$= 0$ , where  $\hat{\mathbf{r}}$  is the unit vector along OA.

It is clear also by symmetry that the three forces will sum to zero. Suppose that the resultant force was non-zero but in some direction. Consider what would happen if the system was rotated through  $60^\circ$  about O.

### Ncert solved problem 3

**Example 1.7** Consider the charges  $q$ ,  $q$ , and  $-q$  placed at the vertices of an equilateral triangle, as shown in Fig. 1.10. What is the force on each charge?

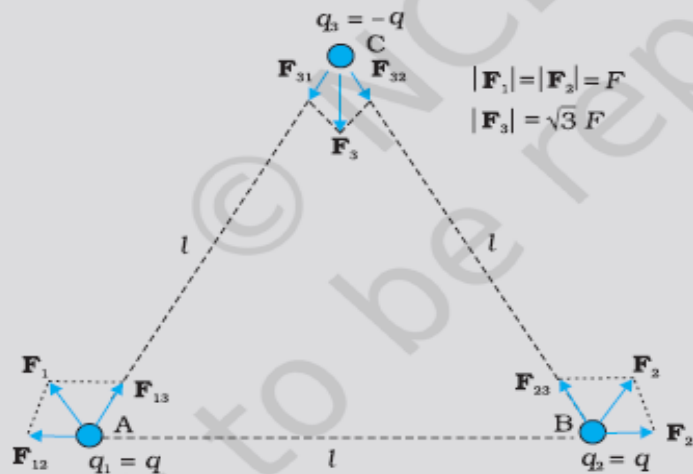


FIGURE 1.10

**Solution** The forces acting on charge  $q$  at A due to charges  $q$  at B and  $-q$  at C are  $\mathbf{F}_{12}$  along BA and  $\mathbf{F}_{13}$  along AC respectively, as shown in Fig. 1.10. By the parallelogram law, the total force  $\mathbf{F}_1$  on the charge  $q$  at A is given by

$\mathbf{F}_1 = F \hat{\mathbf{r}}_1$  where  $\hat{\mathbf{r}}_1$  is a unit vector along BC.

The force of attraction or repulsion for each pair of charges has the

same magnitude  $F = \frac{q^2}{4\pi\epsilon_0 l^2}$

The total force  $\mathbf{F}_2$  on charge  $q$  at B is thus  $\mathbf{F}_2 = F \hat{\mathbf{r}}_2$ , where  $\hat{\mathbf{r}}_2$  is a unit vector along AC.

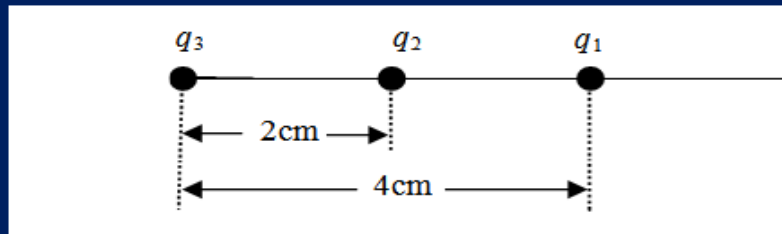
Similarly the total force on charge  $-q$  at C is  $\mathbf{F}_3 = \sqrt{3} F \hat{\mathbf{n}}$ , where  $\hat{\mathbf{n}}$  is the unit vector along the direction bisecting the  $\angle BCA$ .

It is interesting to see that the sum of the forces on the three charges is zero, i.e.,

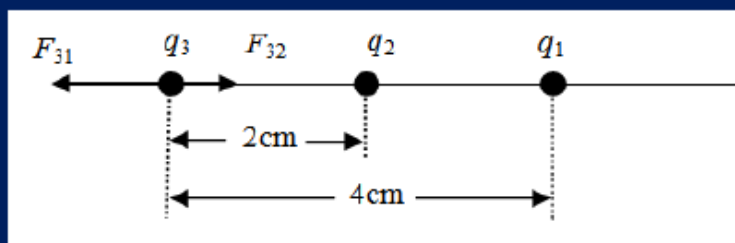
$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$$

The result is not at all surprising. It follows straight from the fact that Coulomb's law is consistent with Newton's third law. The proof is left to you as an exercise.

Two charges are located on the positive x-axis of a coordinate system, as shown in figure below. Charge  $q_1=2\text{nC}$  is 2cm from the origin, and charge  $q_2=-3\text{nC}$  is 4cm from the origin. What is the total force exerted by these two charges on a charge  $q_3=5\text{nC}$  located at the origin?



See the solution



$$F_{31} = \frac{(9 \times 10^9)(2 \times 10^{-9})(5 \times 10^{-9})}{(0.04)^2} = 0.56 \times 10^{-4} \text{ N}$$

$$F_{32} = \frac{(9 \times 10^9)(3 \times 10^{-9})(5 \times 10^{-9})}{(0.02)^2} = 3.37 \times 10^{-4} \text{ N}$$

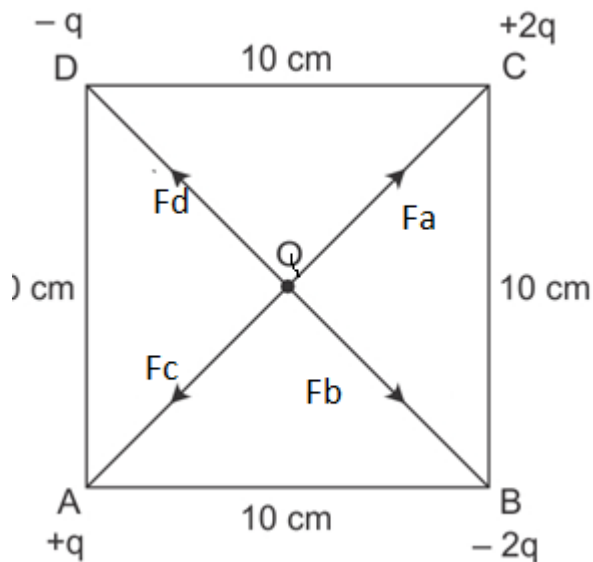
$$F_3 = F_{31} + F_{32}$$

$$\therefore F_3 = 0.56 \times 10^{-4} - 3.37 \times 10^{-4} = -2.81 \times 10^{-4} \text{ N}$$

#### SAMPLE PROBLEM 4

A square has four charges kept at the four corners as shown.

Calculate the net force on the charge of value  $+Q$  kept at the centre.



Since four forces are going to act at Q which is kept at the centre. We will first analyse the direction of the forces.

Charge at A exerts a repulsive force ( $F_a$ ) on Q directed along AC

Charge at B exerts an attractive force ( $F_b$ ) on Q directed along DB

Charge at C exerts a repulsive force ( $F_c$ ) on Q directed along CA

Charge at D exerts an attractive force ( $F_d$ ) on Q directed along BD

Using vectors, we can see that  $F_a$  and  $F_c$  are in opposite direction and will subtract, let their resultant be  $F_1$

Similarly  $F_b$  and  $F_d$  are in opposite direction and will subtract, let their resultant be  $F_2$

Net force of Q is now the vector sum of  $F_1$  and  $F_2$  using parallelogram law of vectors (angle between diagonals is 90 degrees in a square)

$$\mathbf{F_1} = \mathbf{F_c} - \mathbf{F_a} = \frac{kQ2q}{a^2} - \frac{kQq}{a^2} = \frac{kQq}{a^2} \text{ (remember these are only magnitudes, we have already analysed the directions) this is along CA}$$

$$\mathbf{F_2} = \mathbf{F_b} - \mathbf{F_d} = \frac{kQ2q}{a^2} - \frac{kQq}{a^2} = \frac{kQq}{a^2} \text{ this is along DB}$$

$$\text{Net Force is given by } F_{\text{net}} = \sqrt{2} \frac{kQq}{a^2} \text{ along negative y axis}$$

**See more solved problems on the links given below**

[https://www.brainkart.com/article/Coulomb---s-Law---Solved-Example-Problems\\_38359/](https://www.brainkart.com/article/Coulomb---s-Law---Solved-Example-Problems_38359/)

<https://www.problemsphysics.com/problems/electrostatic.html>

<http://www.csun.edu/~rd436460/100B/lectures/chapter19-1-3.pdf>

[http://folk.uio.no/ravi/cutn/elec\\_mag/5\\_problem.pdf](http://folk.uio.no/ravi/cutn/elec_mag/5_problem.pdf)

**Unsolved problems on the topic for additional practice:**

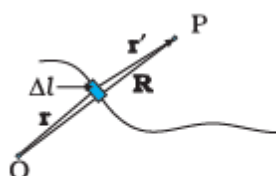
<https://www.crsd.org/cms/lib5/PA01000188/Centricity/Domain/773/D4and5%20Coulombs%20Law%20Worksheet%20with%20reading%20assignment%20attached.pdf>

### **Continuous charge distribution**

A system of a large number of closely spaced charges is known as continuous charge distribution.

A continuous charge distribution may be of the following three types:

1. Linear charge distribution- is a charge distribution in which charge is distributed uniformly along a line that is a charged wire or ring etc.

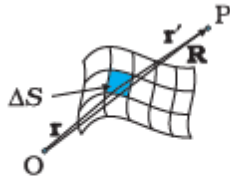


Line charge  $\Delta Q = \lambda \Delta l$

*Diagram courtesy NCERT*

The linear charge density  $\lambda$  of a wire is given as  $dq/dl$  where  $dl$  is a small line element of a wire and  $dQ$  is the charge contained in the line element. The unit for  $\lambda$  is C/m

2 Surface charge distribution- is defined as continuous charge distribution over some area. For example a charged thin sheet.



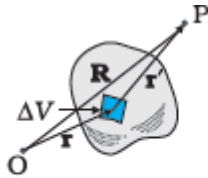
$$\text{Surface charge } \Delta Q = \sigma \Delta S$$

*Diagram courtesy NCERT*

The surface charge density  $\sigma$  of a wire is given as  $dq/ds$  where  $ds$  is a small area element of the sheet and  $dQ$  is the charge contained in the area element.

The unit for  $\sigma$  is  $C/m^2$

3 Volume charge distribution- is defined as continuous charge distribution over a volume. For example a metal sphere or cylinder etc.



$$\text{Volume charge } \Delta Q = \rho \Delta V$$

*Diagram courtesy NCERT*

The volume charge density or simply called charge density  $\rho$  is given as  $dq/dv$  where  $dv$  is a small volume element of the sheet and  $dQ$  is the charge contained in the volume element.

The unit for  $\rho$  is  $C/m^3$

## Video links

### Superposition principle

<https://www.youtube.com/watch?v=S5yICTzMD6Y&t=30s>

[https://www.youtube.com/watch?v=isOF9X\\_NH\\_I](https://www.youtube.com/watch?v=isOF9X_NH_I)



## ASSIGNMENT

1 A charge  $Q$  is positioned on the  $x$ -axis at  $x = a$ . Where should a charge  $Q_2 = -4Q$  be placed to produce a net electrostatic force of zero on a third charge,  $Q_3 = Q$ , located at the origin?

- a) at the origin      b) at  $x = 2a$       c) at  $x = -2a$       d) at  $x = -a$

2 Two point charges are fixed on the  $x$ -axis:  $q_1 = 6.0 \text{ C}$  is located at the origin,  $O$ , with  $x_1 = 0.0 \text{ cm}$ , and  $q_2 = -3.0 \text{ C}$  is located at point  $A$ , with  $x_2 = 8.0 \text{ cm}$ . Where should a third charge,  $q_3$ , be placed on the  $x$ -axis so that the total electrostatic force acting on it is zero?

- a) 19 cm   b) 27 cm   c) 0.0 cm   d) 8.0 cm   e) -19 cm

3 Eight  $1 \text{ C}$  charges are arrayed along the  $y$ -axis located every  $2 \text{ cm}$  starting at  $y = 0$  and extending to  $y = 14 \text{ cm}$ . Find the force on the charge at  $y = 4 \text{ cm}$ .

4 Four charges, each equal to  $-Q$ , are placed at the corners of a square and a charge  $+q$  is placed at its centre. If the system is in equilibrium, the value of  $q$  is

(A)  $\frac{Q}{4}(1 + 2\sqrt{2})$       (B)  $-\frac{Q}{4}(1 + 2\sqrt{2})$

(C)  $-\frac{Q}{2}(1 + 2\sqrt{2})$       (D)  $\frac{Q}{2}(1 + 2\sqrt{2})$

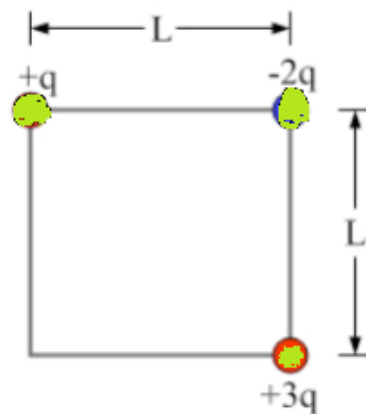
5 Four charges  $2 \times 10^{-7} \text{ C}$ ,  $-2 \times 10^{-7} \text{ C}$ ,  $-1 \times 10^{-7} \text{ C}$  and  $1 \times 10^{-7} \text{ C}$  are placed at the corners  $A$ ,  $B$ ,  $C$  and  $D$  of a square of side  $5 \text{ cm}$  respectively. Find the force on the charge placed at  $A$ .

7 It is required to hold four equal point charges  $+q$  in equilibrium at the corners of a square. Find the point charge that will do this if placed at the center of the square.

8 Two point charges of values  $q$  and  $2q$  are kept a distance  $d$  apart in air. A third charge  $Q$  is to be kept along the line joining the two charges in such a way that the net force acting on  $q$  and  $2q$  is zero. Calculate the position of charge  $Q$  in terms of  $q$  and  $d$ .

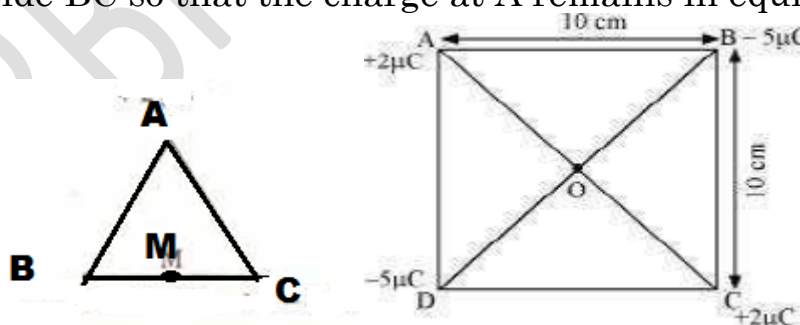
9 Charges  $+5\mu\text{C}$ ,  $+10\mu\text{C}$  and  $-10\mu\text{C}$  are placed in air at the corners A, B and C respectively of an equilateral triangle ABC of side 5 cm. Determine the resultant force on the charge at A.

10 Three balls, with charges of  $+q$ ,  $-2q$ , and  $+3q$ , are placed at the corners of a square measuring  $L$  on each side, as shown. What is the magnitude and direction of the net electrostatic force on the ball of charge  $+q$ ?



11 Charges  $+5\mu\text{C}$ ,  $+10\mu\text{C}$  and  $-10\mu\text{C}$  are placed in air at the corners A, B and C respectively of an equilateral triangle ABC of side 5 cm. Determine the resultant force on the charge at A.

12 There point charges of  $2\mu\text{C}$ ,  $-3\mu\text{C}$  and  $-3\mu\text{C}$  are kept at the vertices, A, B and C respectively of an equilateral triangle of side 20 cm as shown in the figure. What should be the sign and magnitude of the charge to be placed at the mid-point (M) of side BC so that the charge at A remains in equilibrium?



13 Four point charges  $q_A = 2\mu\text{C}$ ,  $q_B = -5\mu\text{C}$ ,  $q_C = 2\mu\text{C}$ , and  $q_D = -5\mu\text{C}$  are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of  $1\mu\text{C}$  placed at the centre of the square?